Introduction to the Physics of Saturation

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Outline

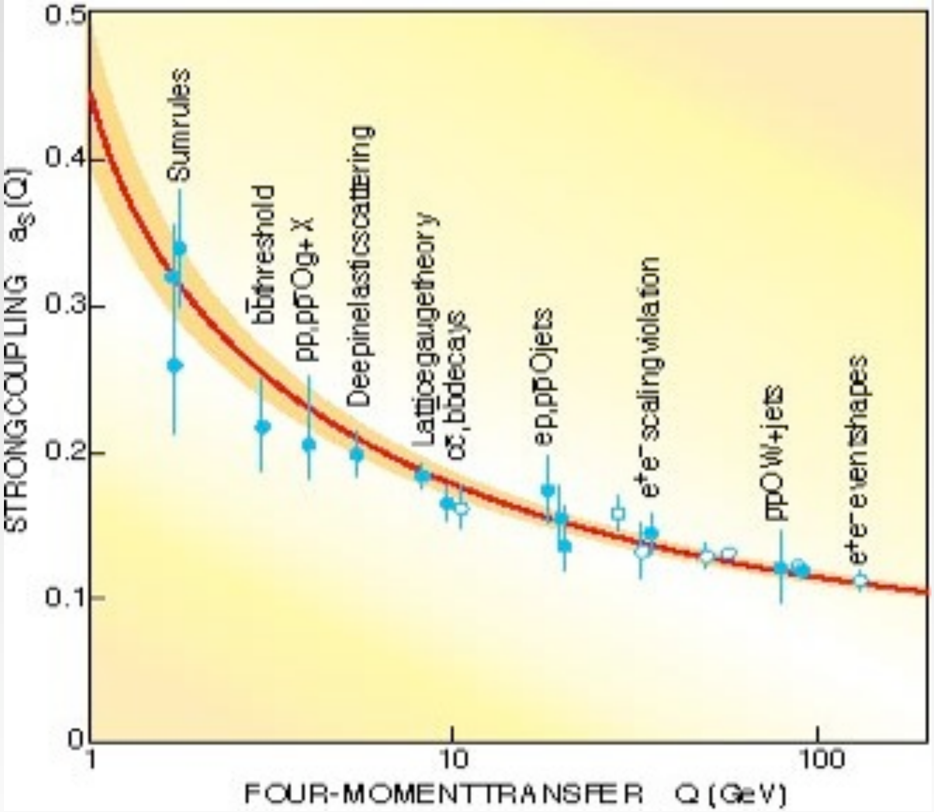
- Preamble
- Review of saturation physics/CGC.
  - Classical fields
  - Quantum evolution
- More recent progress at small-x
  - Running coupling corrections
  - NLO BFKL/BK/JIMWLK corrections
- Conclusions

Preamble

Running of QCD Coupling Constant

⇨ QCD coupling constant $\alpha_S = \dfrac{g^2}{4\pi}$ changes with the momentum scale involved in the interaction

$$\alpha_S = \alpha_S(Q)$$



Asymptotic Freedom!

Gross and Wilczek, Politzer, ca '73

Physics Nobel Prize 2004!

For short distances x < 0.2 fm, or, equivalently, large momenta k > 1 GeV the QCD coupling is small $\alpha_S << 1$ and interactions are weak.

A Question

- Can we understand, qualitatively or even quantitatively, the structure of hadrons and their interactions in High Energy Collisions?
  - What are the total cross sections?
  - What are the multiplicities and production cross sections?
  - Diffractive cross sections.
  - Particle correlations.

What sets the scale of running QCD coupling in high energy collisions?

- "String theorist": $\alpha_S = \alpha_S\left(\sqrt{s}\right) << 1$

  (not even wrong)

- Pessimist: $\alpha_S = \alpha_S\left(\Lambda_{QCD}\right) \sim 1$ we simply can not tackle high energy scattering in QCD.

- pQCD expert: only study high-p_T particles such that $\alpha_S = \alpha_S\left(p_T\right) << 1$

What sets the scale of running QCD coupling in high energy collisions?

- Saturation physics is based on the existence of a large internal mometum scale Q_S which grows with both energy s and nuclear atomic number A

$$\boxed{Q_S^2 \sim A^{1/3}\ s^{\lambda}}$$

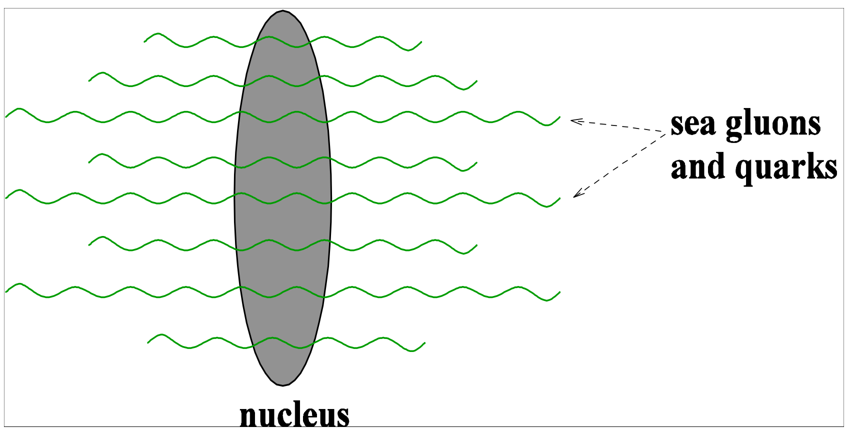
such that $$\boxed{\alpha_S = \alpha_S(Q_S) << 1}$$

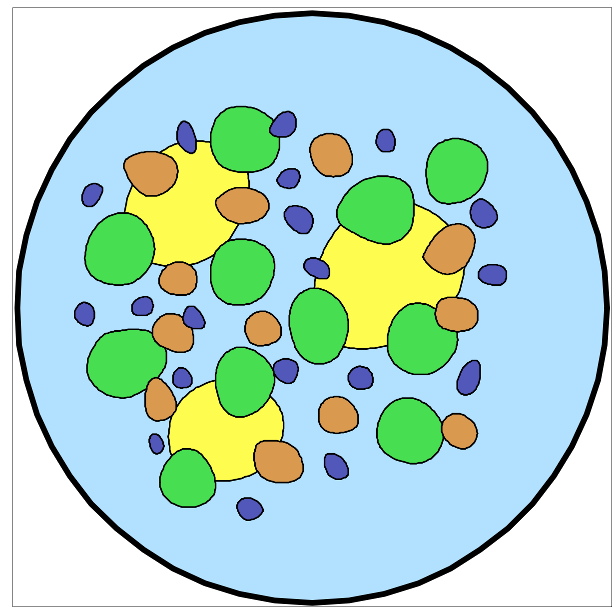
and we can calculate total cross sections, particle spectra and multiplicities, etc from <u>first principles</u>.

Classical Fields

McLerran-Venugopalan Model

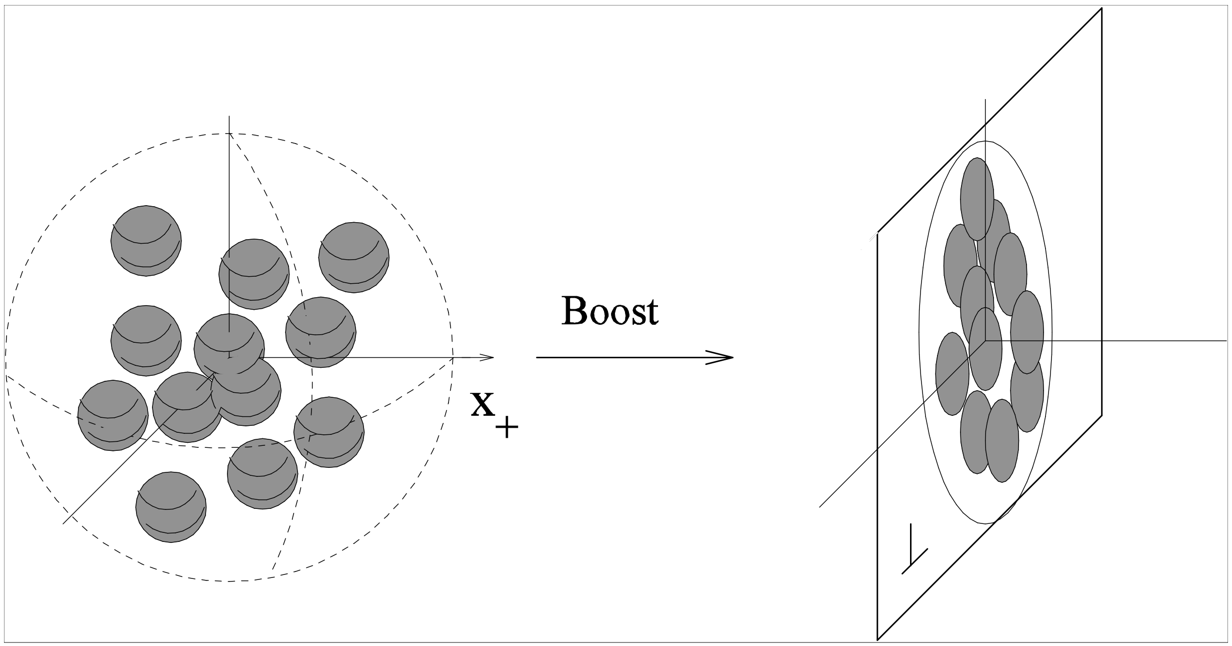
- The wave function of a single nucleus has many small-x quarks and gluons in it.
- In the transverse plane the nucleus is densely packed with gluons and quarks.





Large occupation number ⇨ Classical Field

McLerran-Venugopalan Model



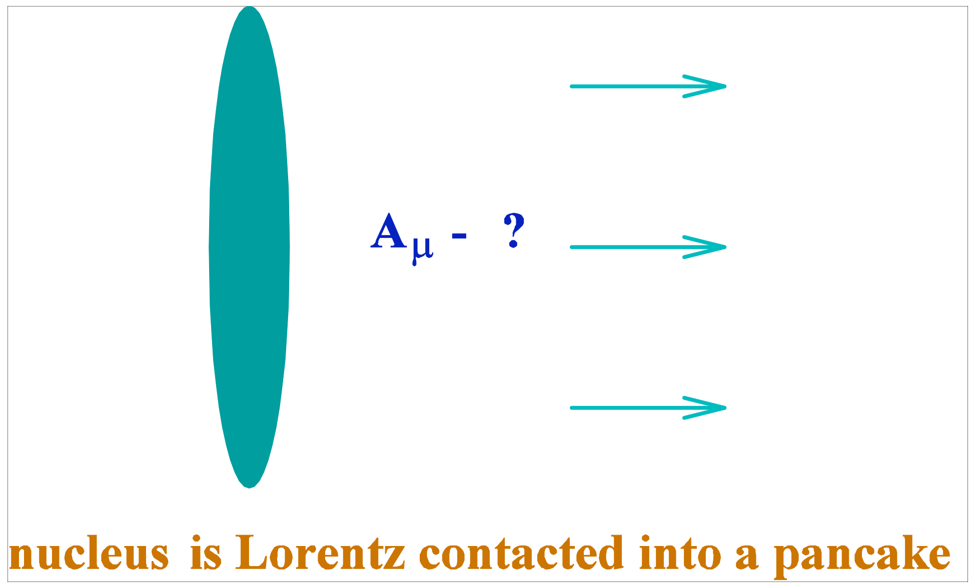
- Large parton density gives a large momentum scale Q_s (the saturation scale).
- For $Q_s >> \Lambda_{QCD}$, get a theory at weak coupling $\alpha_S(Q_S^2) << 1$ and the leading gluon field is <u>classical</u>.

McLerran, Venugopalan '93-'94

McLerran-Venugopalan Model

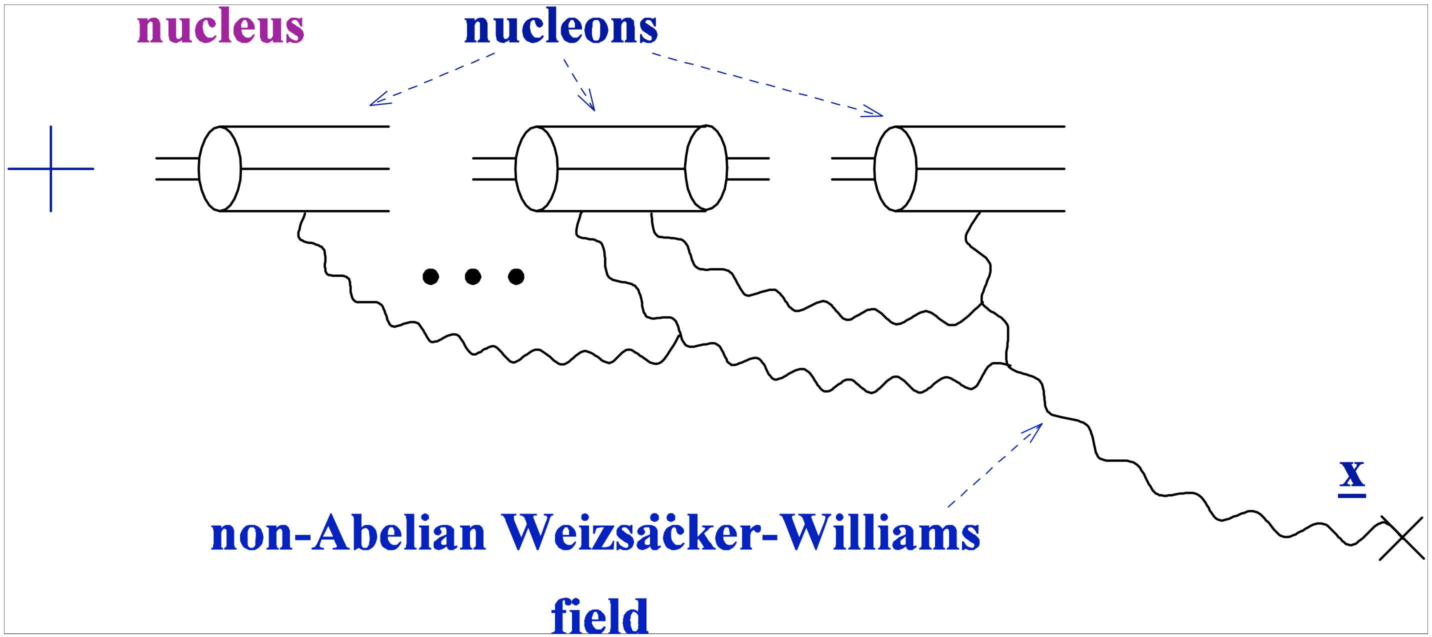
- To find the classical gluon field A_μ of the nucleus one has to solve the non-linear analogue of Maxwell equations – the Yang-Mills equations, with the nucleus as a source of the color charge:

$$D_\nu F^{\mu\nu} = J^\mu$$



Yu. K. '96; J. Jalilian-Marian et al, '96

Classical Field of a Nucleus

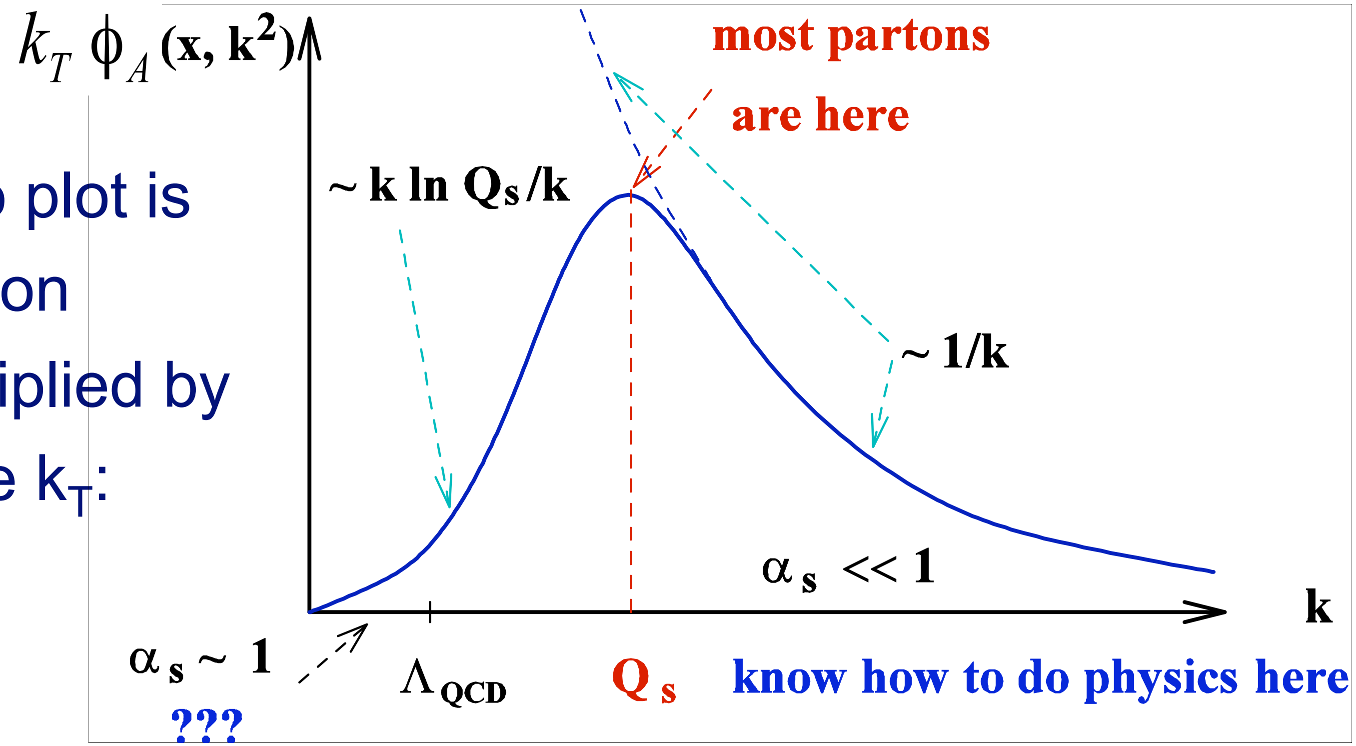


Here's one of the diagrams showing the non-Abelian gluon field of a large nucleus.

The resummation parameter is $\alpha_S^2 A^{1/3}$, corresponding to two gluons per nucleon approximation.

Classical Gluon Distribution

A good object to plot is the classical gluon distribution multiplied by the phase space k_T:



⇨ Most gluons in the nuclear wave function have transverse momentum of the order of $k_T \sim Q_S$ and $Q_S^2 \sim A^{1/3}$

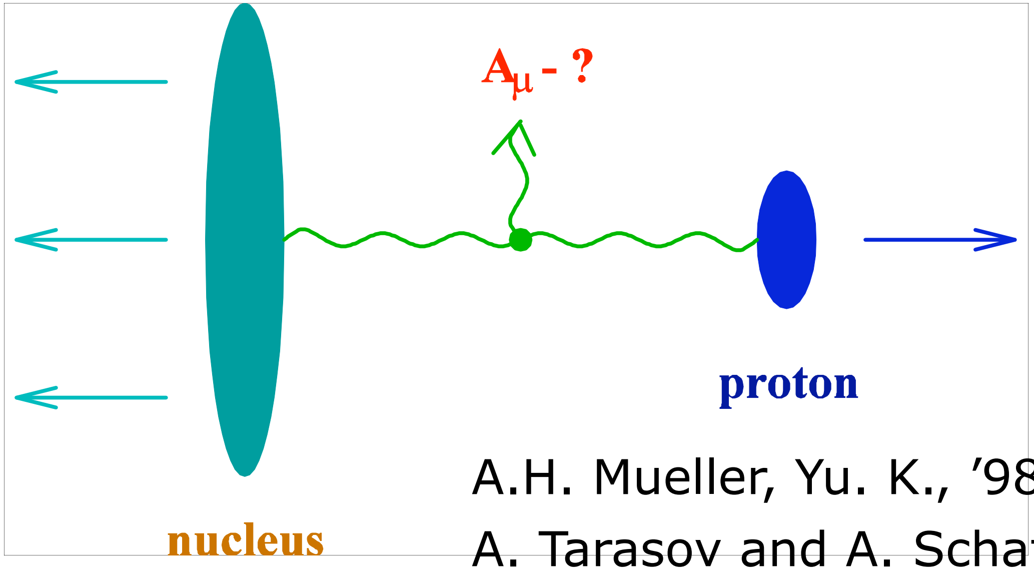
⇨ We have a small coupling description of the whole wave function in the classical approximation.

Classical Gluon Production in Proton-Nucleus Collisions (pA)

To find the gluon production cross section in pA one has to solve the same classical Yang-Mills equations

$$D_\nu F^{\mu\nu} = J^\mu$$

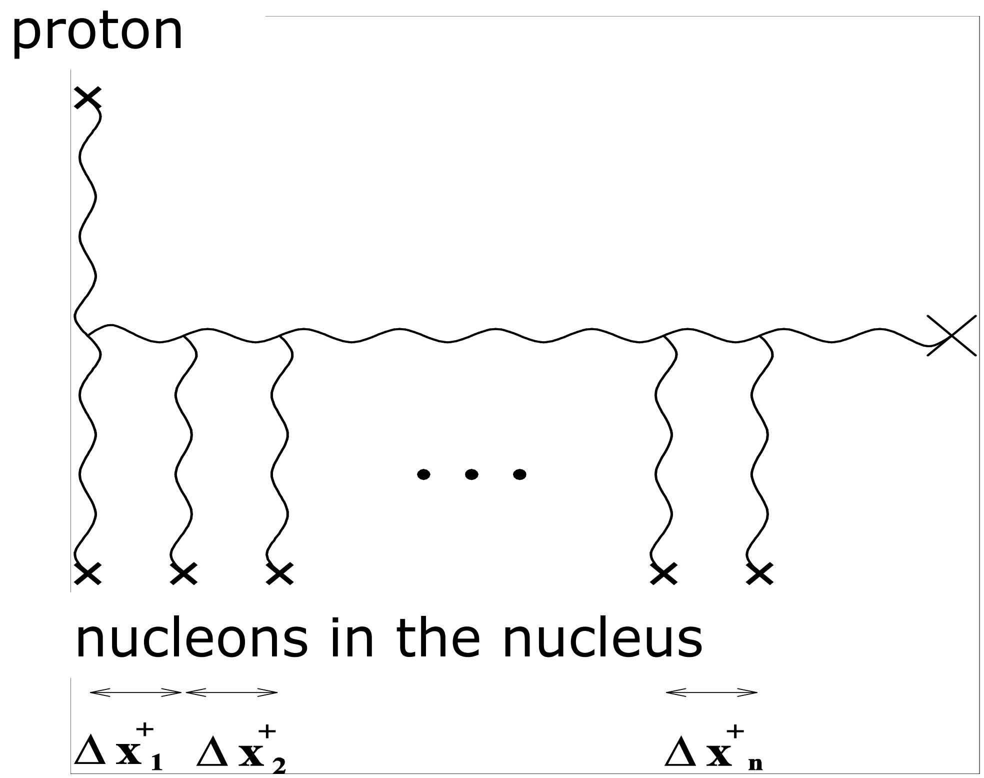
for two sources – proton and nucleus.



A.H. Mueller, Yu. K., '98; B. Kopeliovich, A. Tarasov and A. Schafer, '98; A. Dumitru, L. McLerran '01.

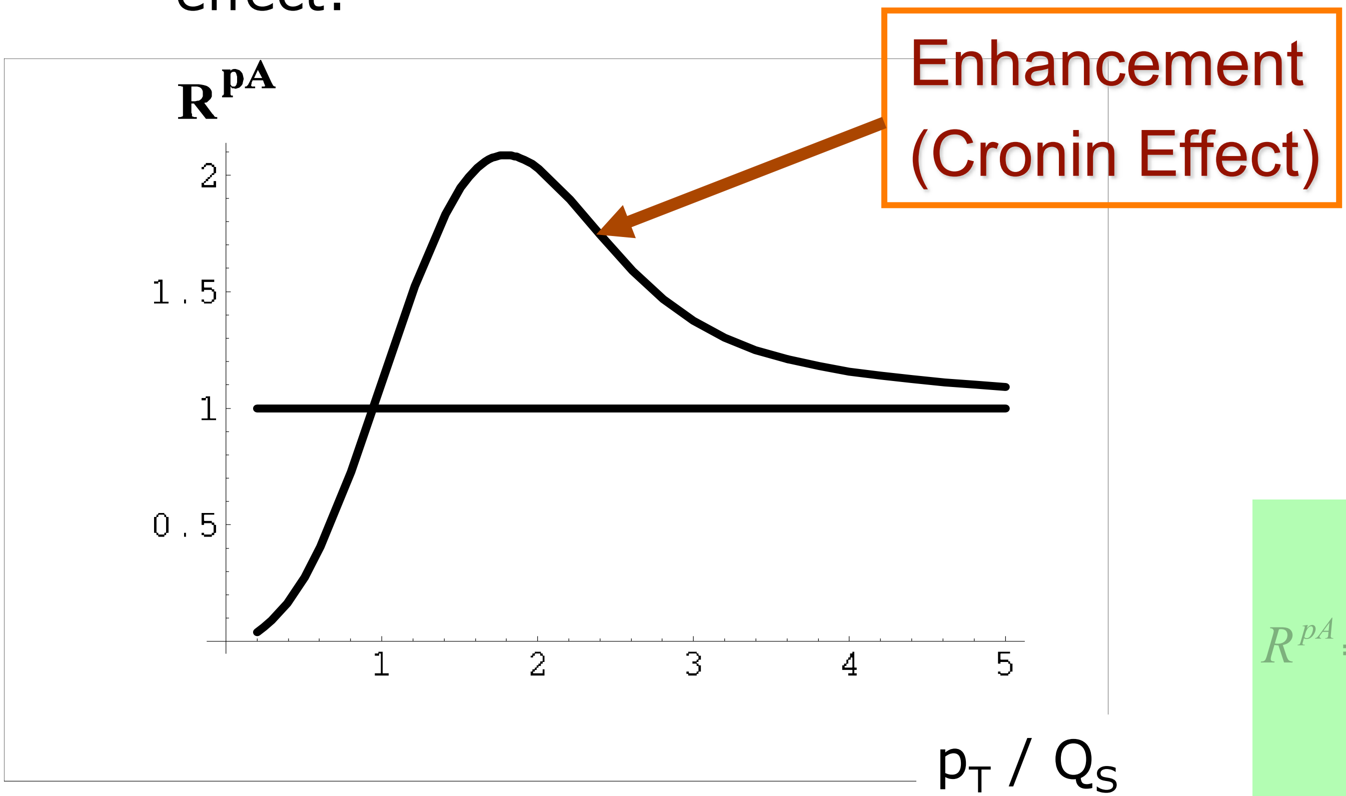
CGC in pA: the diagrams

- Again classical gluon fields correspond to tree-level (no loops) gluon production diagrams:



Classical gluon field in pA: Cronin effect

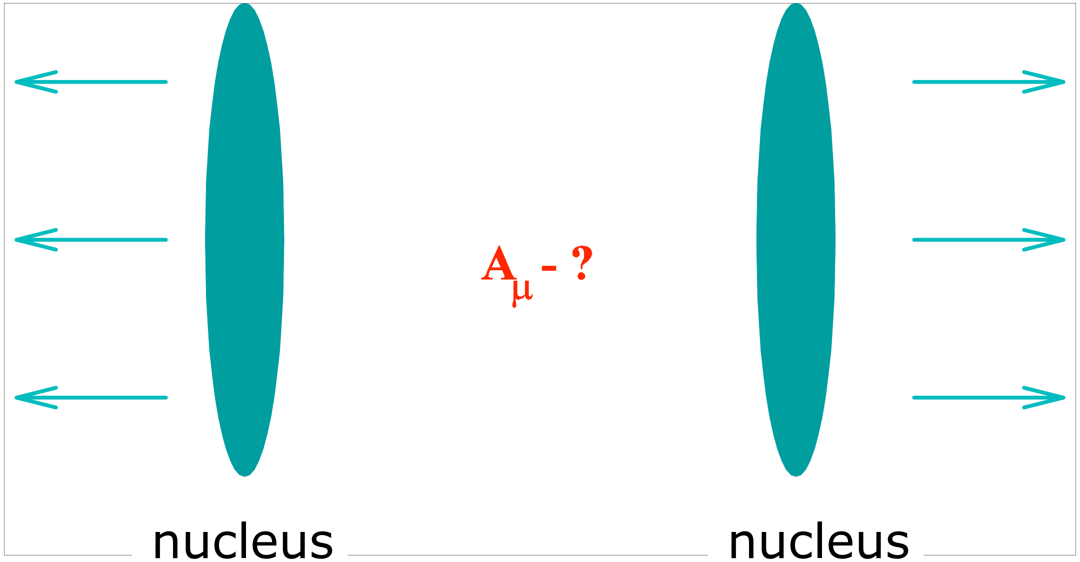
- Classical CGC gluon production in pA lead to Cronin effect:



$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k\,dy}}{A\frac{d\sigma^{pp}}{d^2k\,dy}}$$

Multiple rescatterings -> p_T boradening.

Heavy Ion Collisions in CGC: Classical Gluon Field



- To construct initial conditions for quark-gluon plasma formation in McLerran-Venugopalan model one has to find the classical gluon field left behind by the colliding nuclei.
- No analytical solution exists.
- Perturbative calculations by Kovner, McLerran, Weigert; Rischke, Yu.K.; Gyulassy, McLerran; Balitsky.
- Numerical simulations by Krasnitz, Nara and Venugopalan, and by Lappi, and an analytical ansatz by Yu. K for full solution.

Quantum Evolution

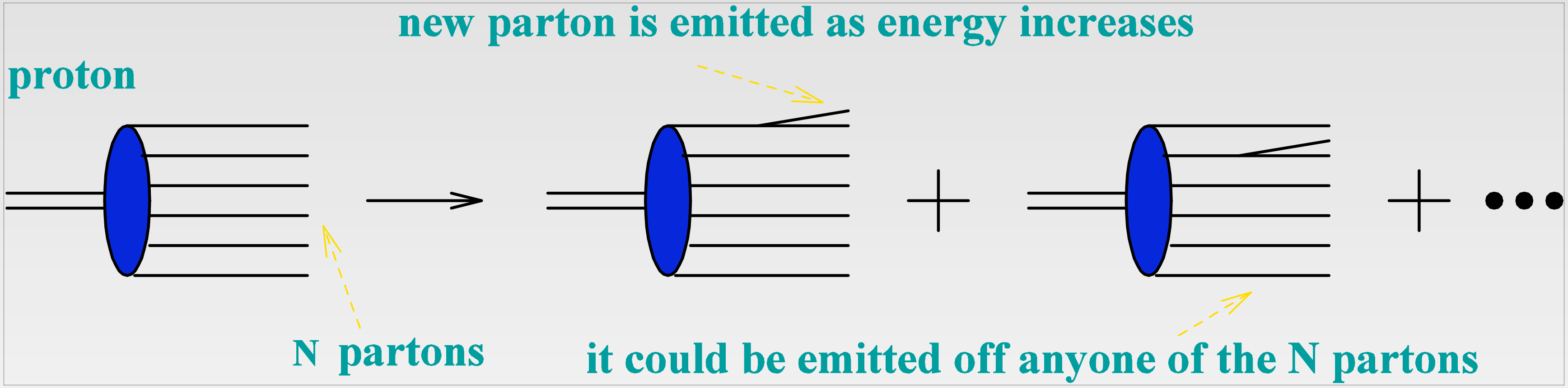
Why Evolve?

- No energy or rapidity dependence in classical field and resulting cross sections.
- Energy/rapidity-dependence comes in through quantum corrections.
- Quantum corrections are included through "evolution equations".

BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov '78

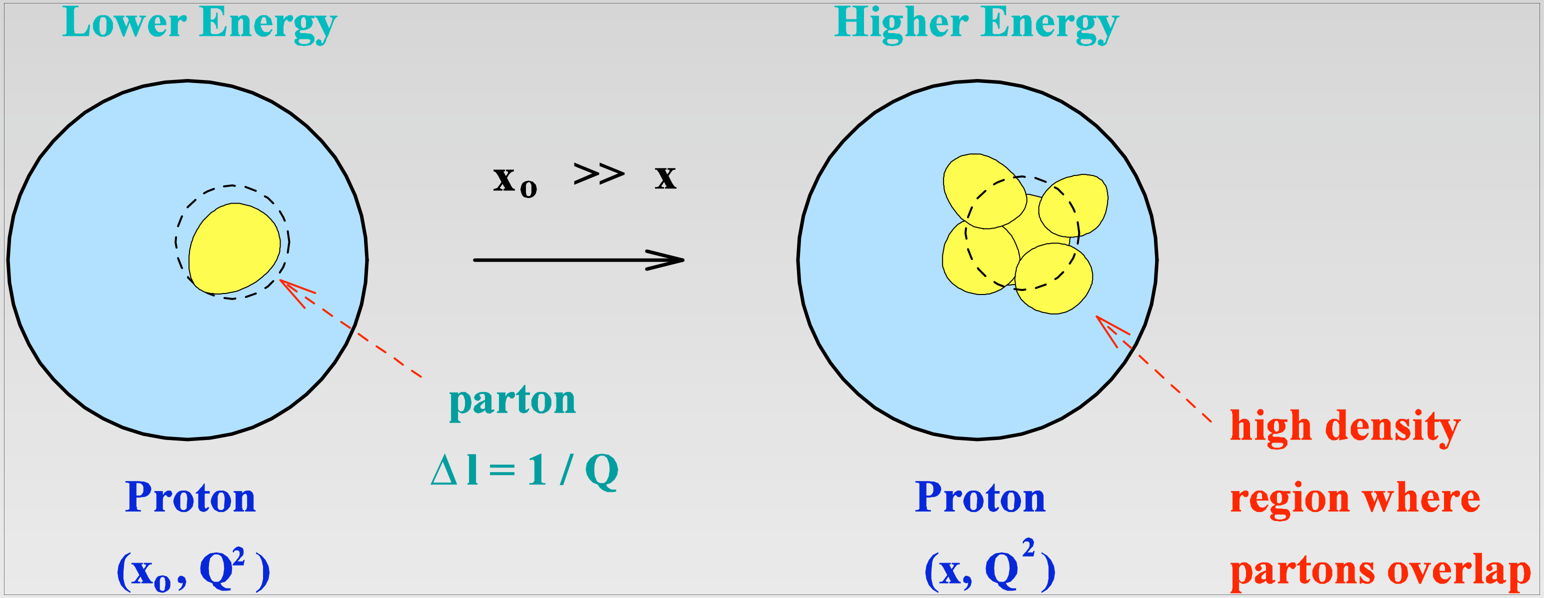
Start with N particles in the proton's wave function. As we increase the energy a new particle can be emitted by either one of the N particles. The number of newly emitted particles is proportional to N.



The BFKL equation for the number of partons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x,Q^2) = \alpha_S K_{BFKL} \otimes N(x,Q^2)$$

BFKL Equation as a High Density Machine



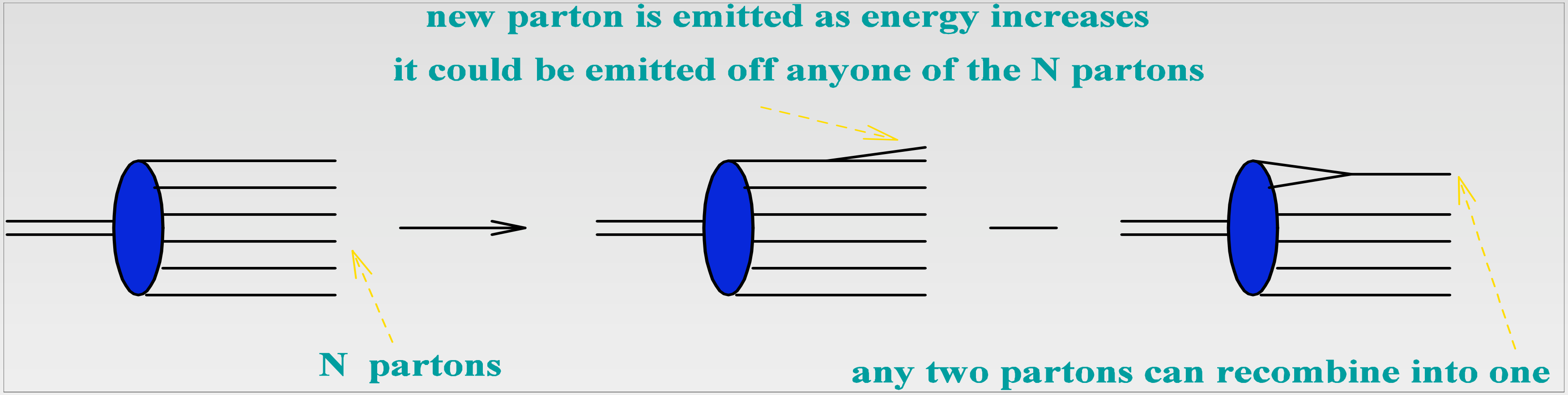
- As energy increases BFKL evolution produces more partons, roughly of the same size. The partons overlap each other creating areas of very high density.
- Number density of partons, along with corresponding cross sections grows as a power of energy

$$N \sim s^{\Delta}$$

$$\sigma_{total} \leq 2\pi R^2$$

Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.



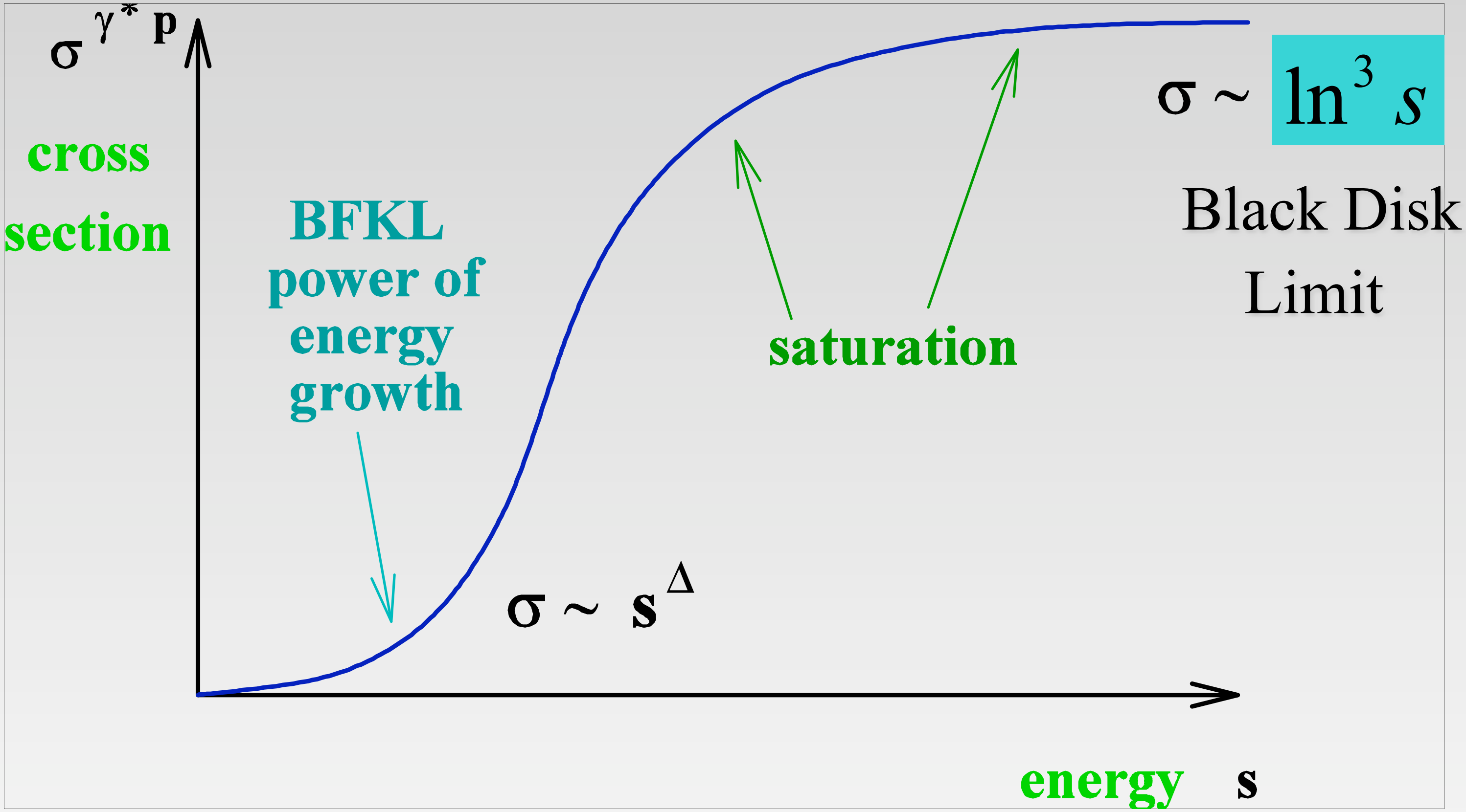
$$\frac{\partial N(x,k^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,k^2) - \alpha_s [N(x,k^2)]^2$$

Number of parton pairs ~ N^2

Yu. K. '99 (large N_C QCD)

I. Balitsky '96 (effective lagrangian)

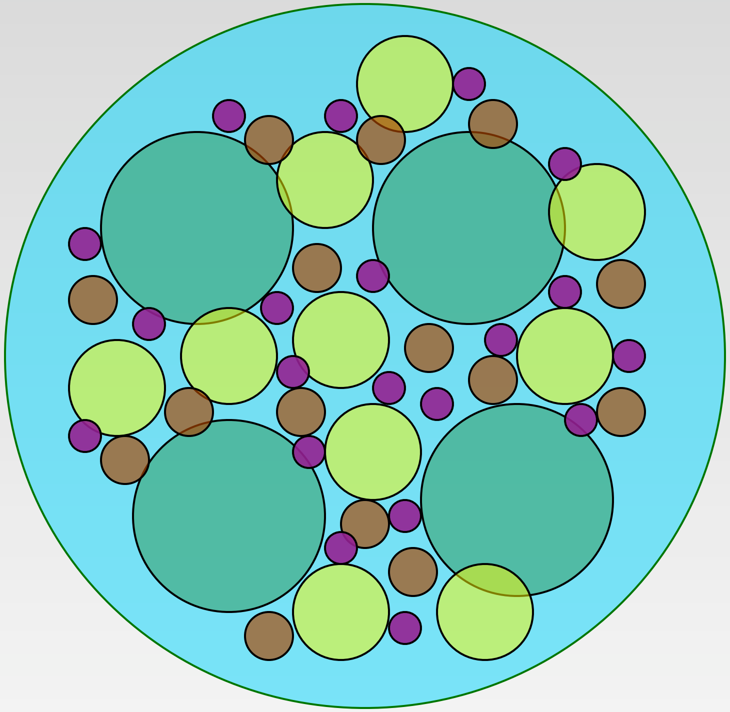
Nonlinear Equation: Saturation



Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

Nonlinear Evolution at Work

Proton



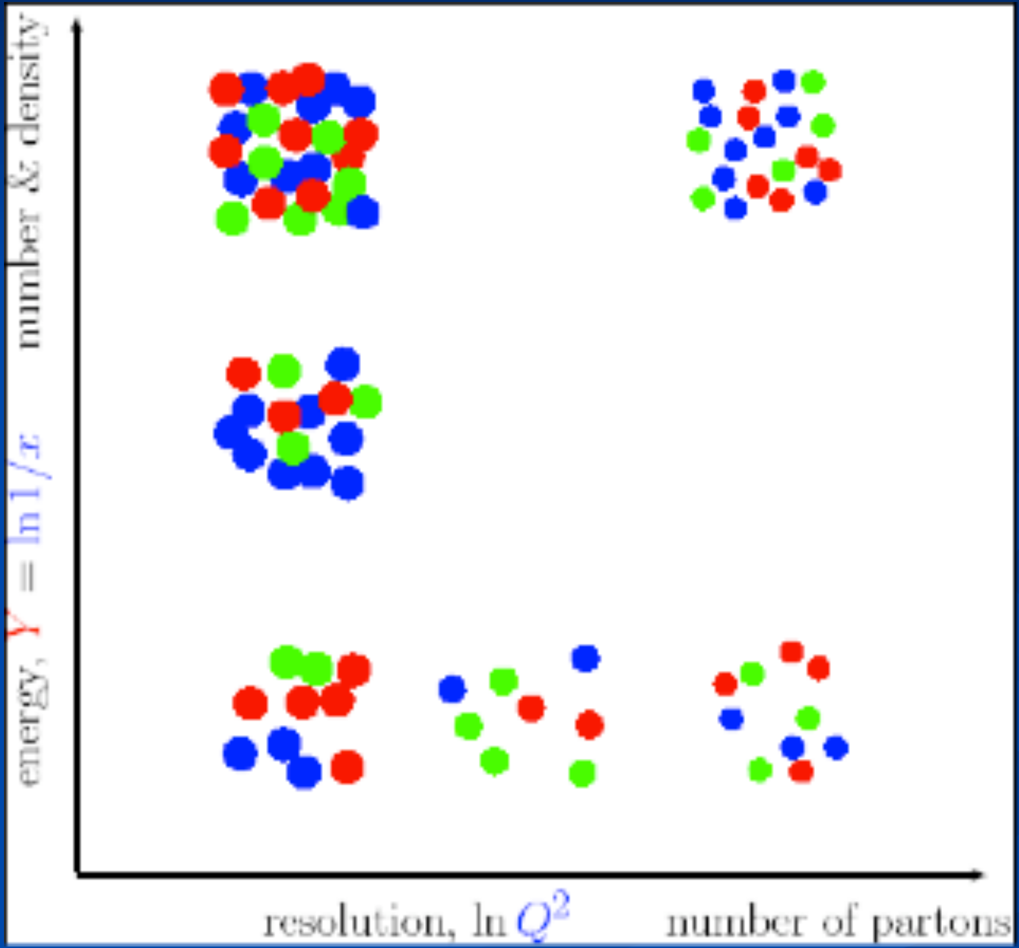
✓ First partons are produced overlapping each other, all of them about the same size.

✓ When some critical density is reached no more partons of given size can fit in the wave function. The proton starts producing smaller partons to fit them in.

Color Glass Condensate

Map of High Energy QCD

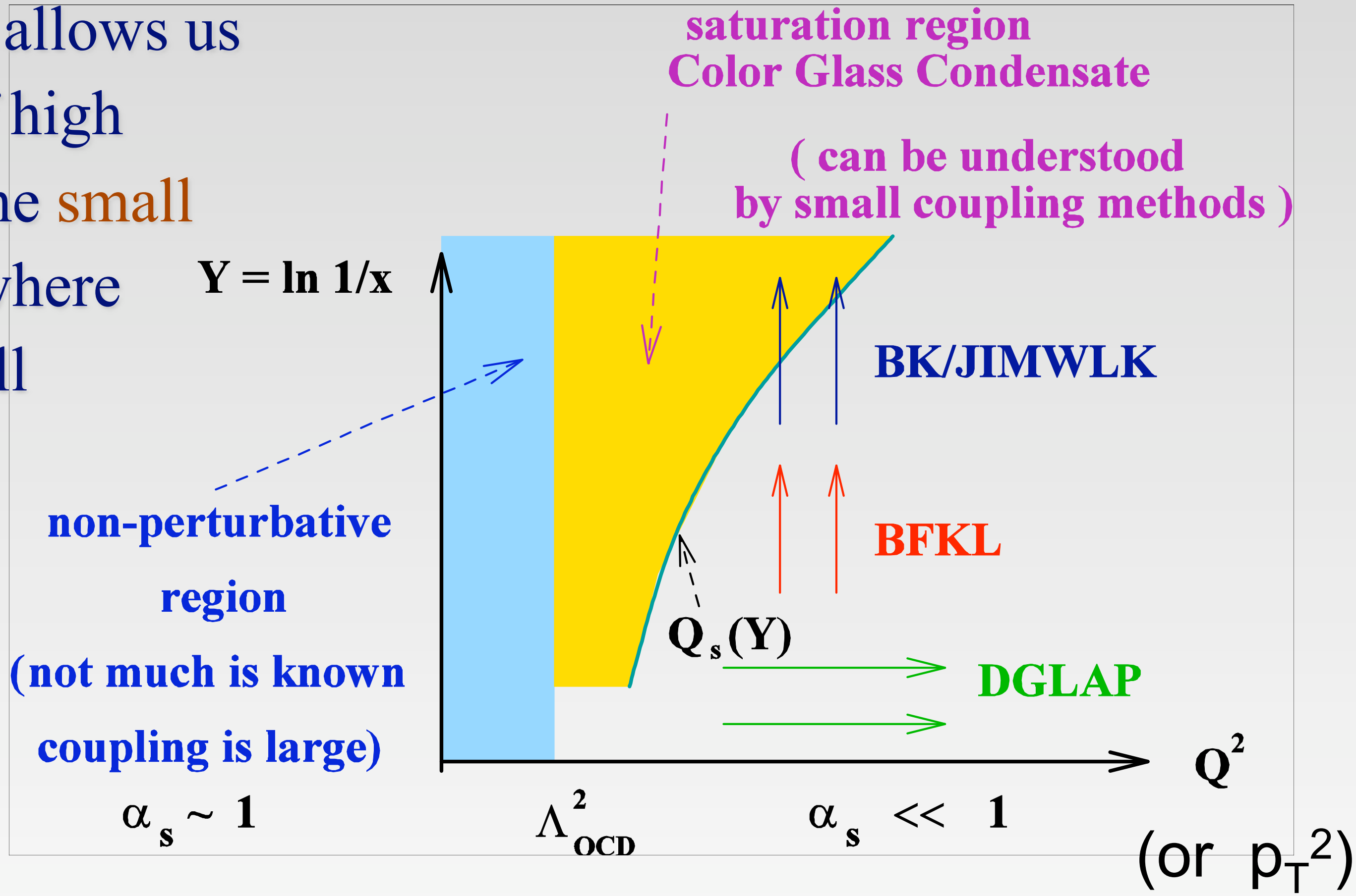
energy



size of gluons

Map of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!



Transition to saturation region is characterized by the saturation scale

$$\boxed{Q_S^2 \sim A^{1/3} \left(\frac{1}{x}\right)^{\lambda}}$$

Going Beyond Large N_C: JIMWLK

To do calculations beyond the large-N_C limit on has to use a functional integro-differential equation written by Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran and Weigert (JIMWLK):

$$\boxed{\frac{\partial Z}{\partial Y} = \alpha_S \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho(u)\delta\rho(v)} [Z\chi(u,v)] - \frac{\delta}{\delta\rho(u)} [Z\sigma(u)] \right\}}$$

where the functional Z[ρ] can then be used for obtaining wave function-averaged observables (like Wilson loops for DIS):

$$< O >= \frac{\int D\rho\, Z[\rho] O[\rho]}{\int D\rho\, Z[\rho]}$$

Going Beyond Large N_C: JIMWLK

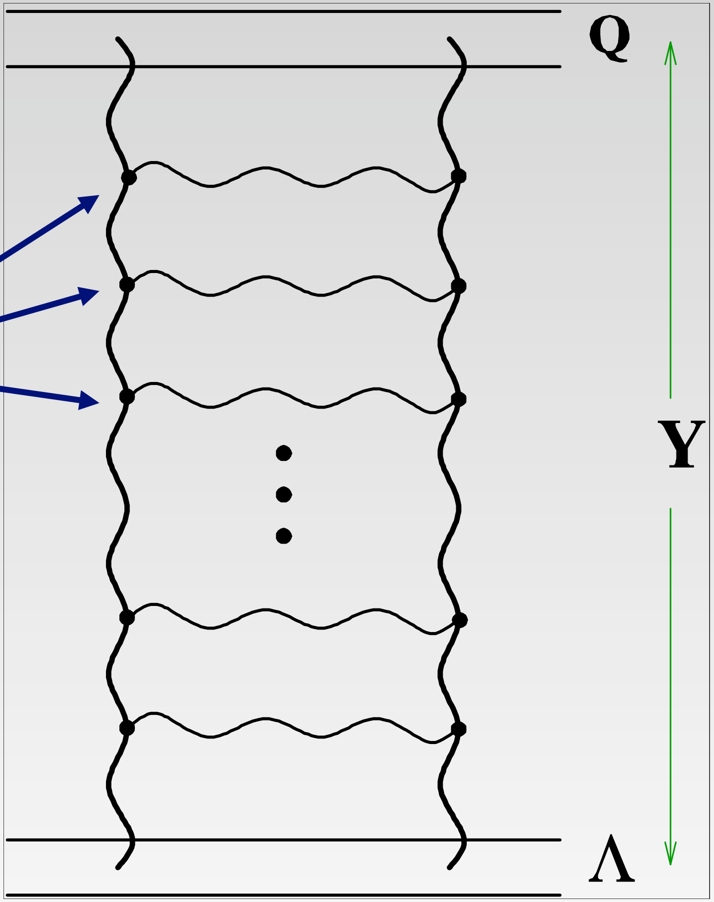
- The JIMWLK equation has been solved on the lattice by K. Rummukainen and H. Weigert

- For the dipole amplitude $N(x_0,x_1, Y)$, the **relative** corrections to the large-N_C limit BK equation are **< 0.001 !** Not the naïve $1/N_C^2 \sim 0.1$!

- The reason for that is dynamical, and is largely due to saturation effects suppressing the bulk of the potential $1/N_C^2$ corrections (Yu.K., J. Kuokkanen, K. Rummukainen, H. Weigert, '08).

BFKL Equation

In the conventional Feynman diagram picture the BFKL equation can be represented by a ladder graph shown here. Each rung of the ladder brings in a power of α ln s.



The resulting dipole amplitude grows as a power of energy

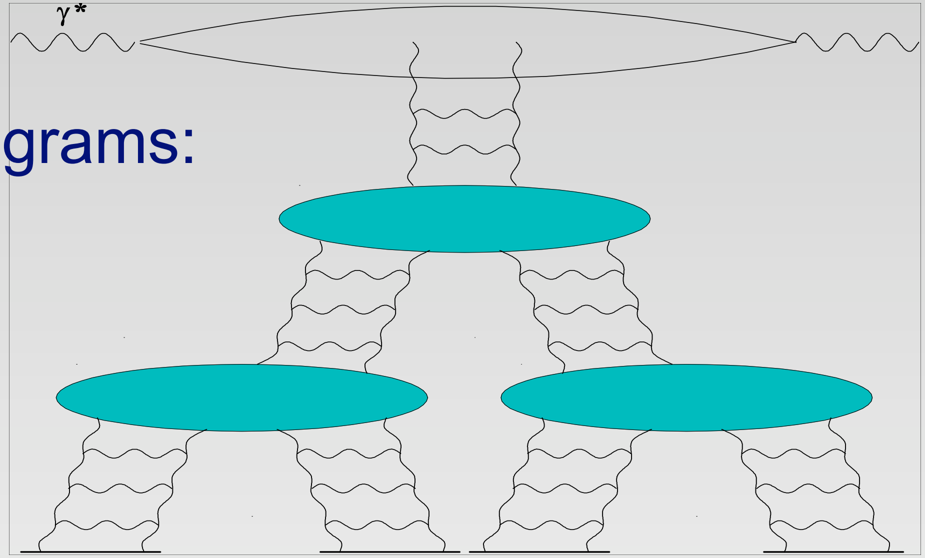
$$N \sim s^{\Delta}$$

violating Froissart unitarity bound

$$\sigma_{tot} \leq const \ln^2 s$$

GLR-MQ Equation

Gribov, Levin and Ryskin ('81) proposed summing up "fan" diagrams:



Mueller and Qiu ('85) summed "fan" diagrams for large Q^2.

The GLR-MQ equation reads:

$$\frac{\partial \phi(x,k^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes \left[\phi(x,k^2) - \alpha_s [\phi(x,k^2)]^2 \right]$$

GLR-MQ equation has the same principle of recombination as BK and JIMWLK. GLR-MQ equation was thought about as the first non-linear correction to the linear BFKL evolution. BK/JIMWLK derivation showed that there are no more terms in the large-N_C limit and obtained the correct kernel for the non-linear term (compared to GLR suggestion).

Geometric Scaling

- One of the predictions of the JIMWLK/BK evolution equations is geometric scaling:

  DIS cross section should be a function of one parameter:

$$\sigma_{DIS}(x, Q^2) = \sigma_{DIS}(Q^2 / Q_S^2(x))$$

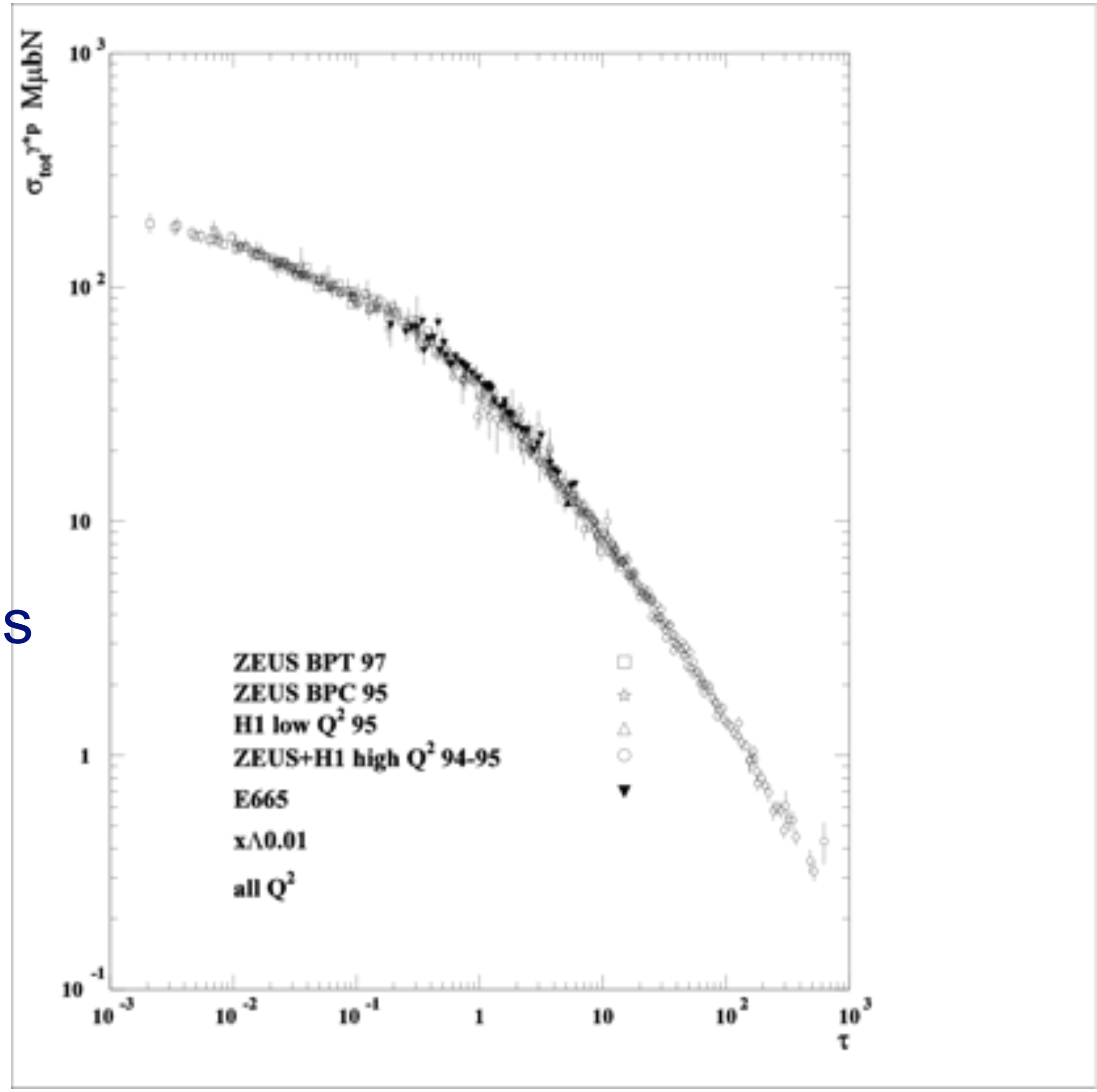
(Levin, Tuchin '99; Iancu, Itakura, McLerran '02)

Geometric Scaling in DIS

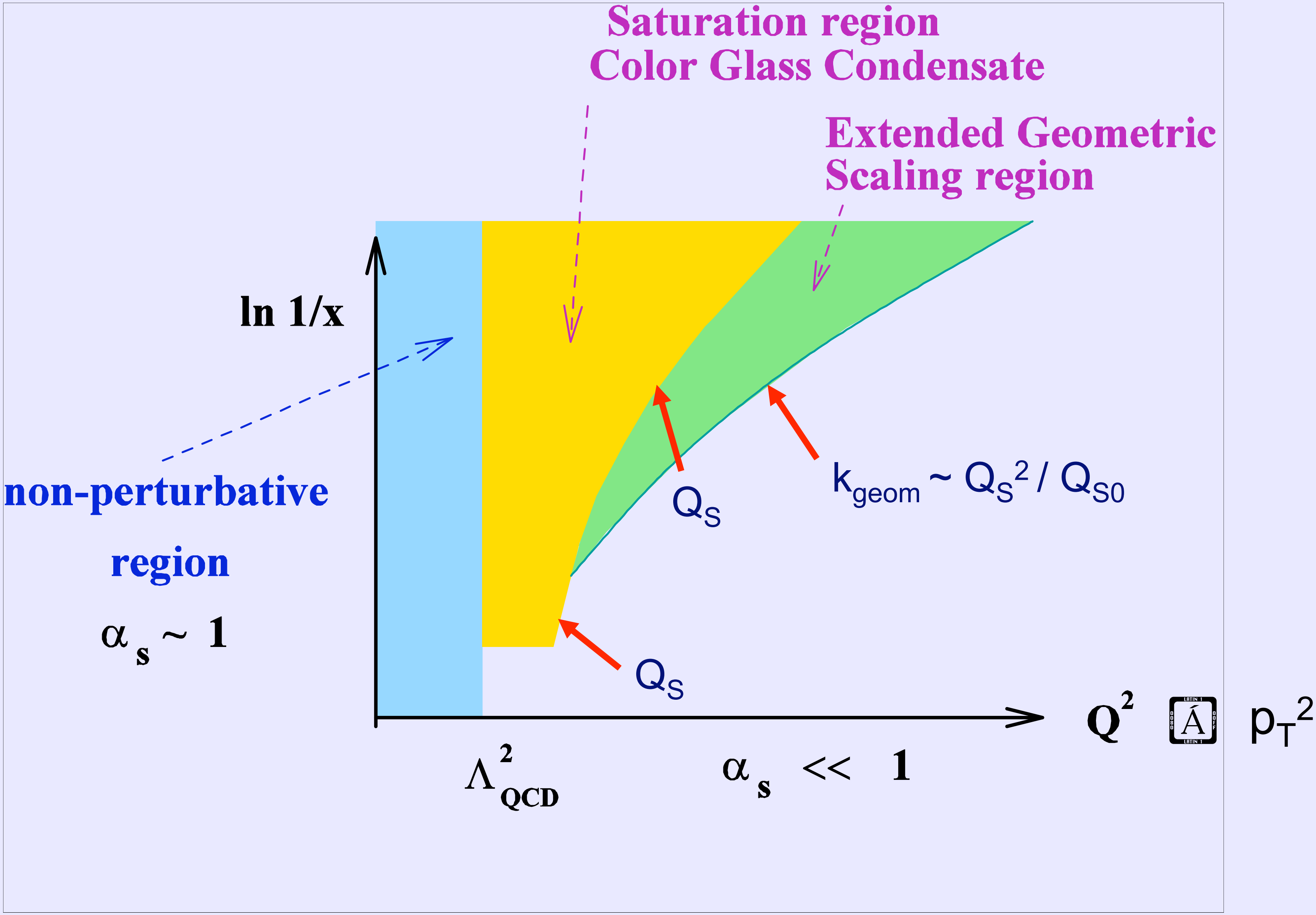
Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in ’00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just <u>one</u> variable:

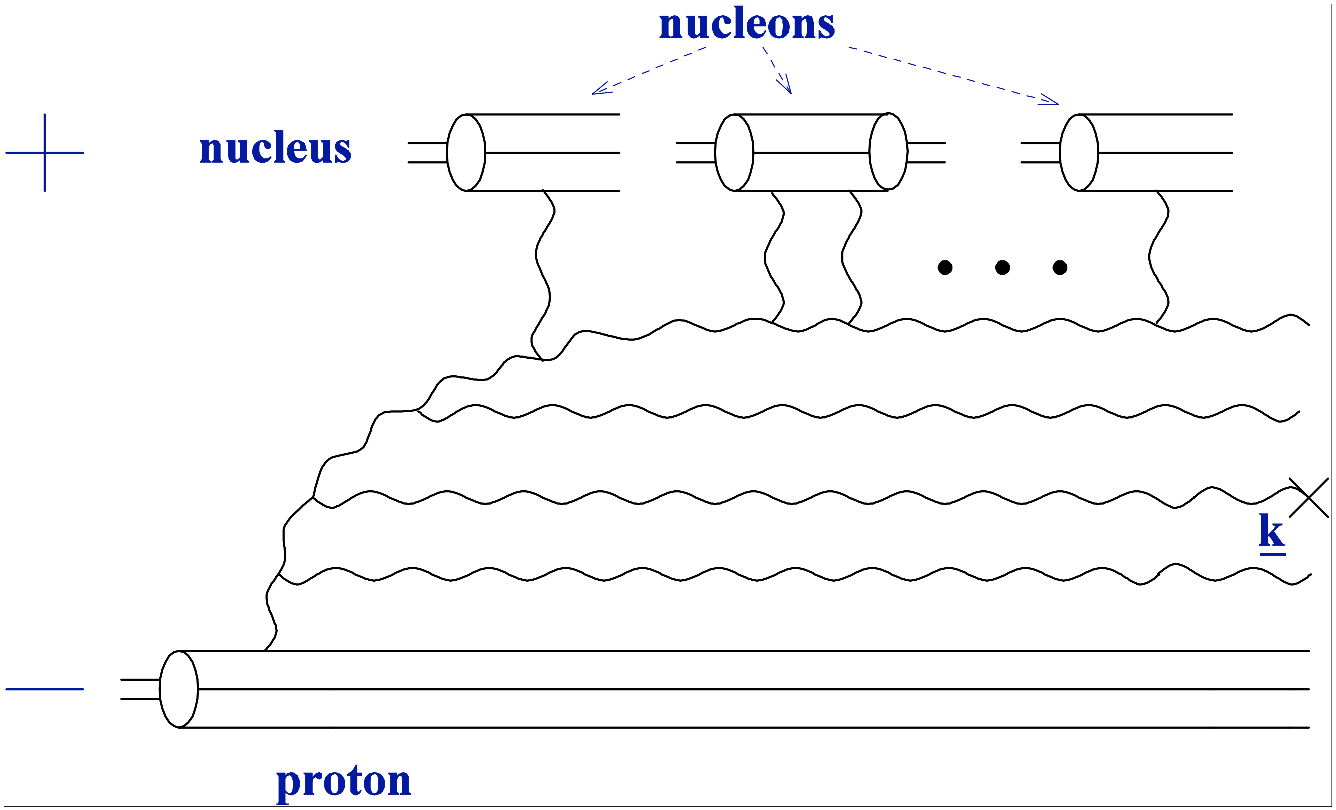
$$\tau = \frac{Q^2}{Q_S^2(x)}$$



Map of High Energy QCD



Quantum Evolution and Particle Production

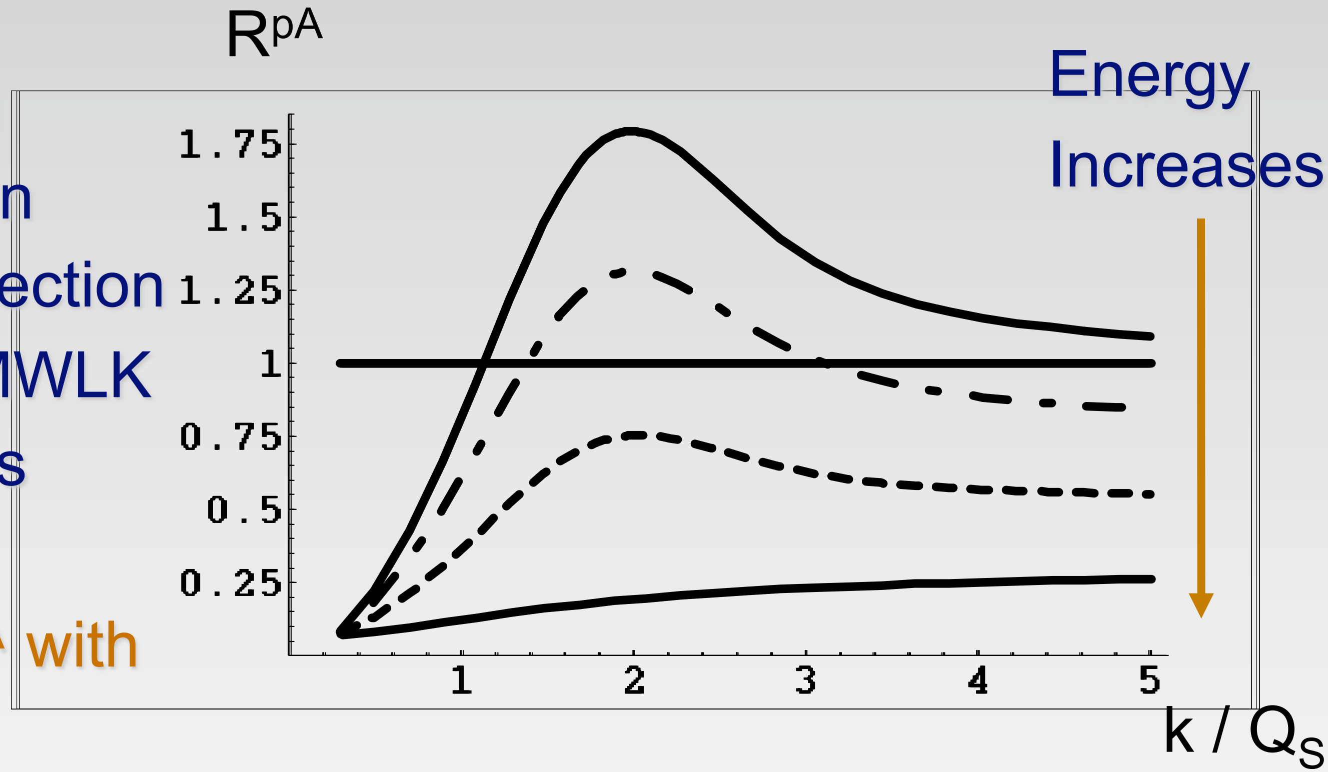


To understand the energy dependence of particle production in pA one needs to include quantum evolution resumming graphs like this one. It resums powers of $\alpha \ln 1/x = \alpha Y$.

(Yu. K., K. Tuchin, '01)

Gluon Production in pA: BK Evolution

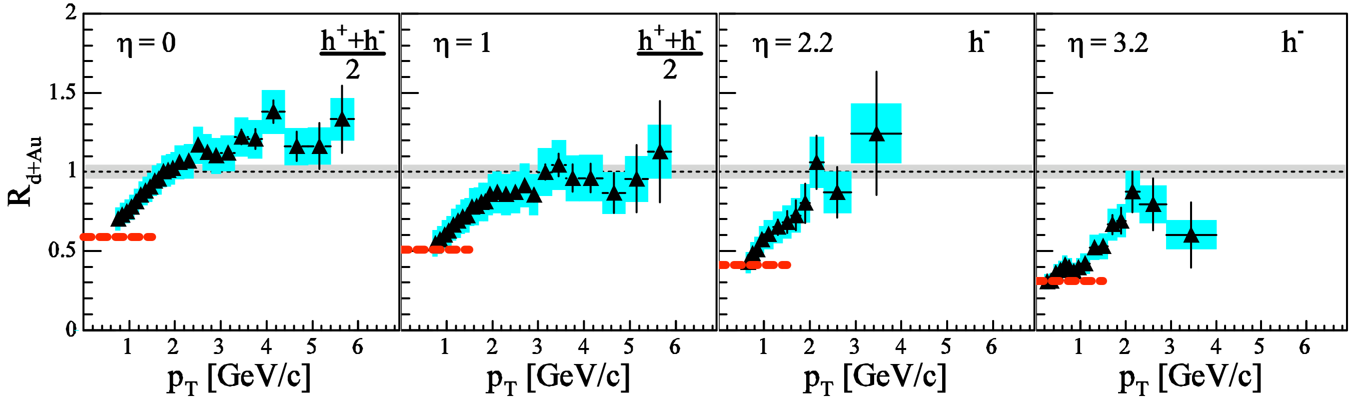
Including quantum corrections to gluon production cross section in pA using BK/JIMWLK evolution equations introduces suppression in R^{pA} with increasing energy!



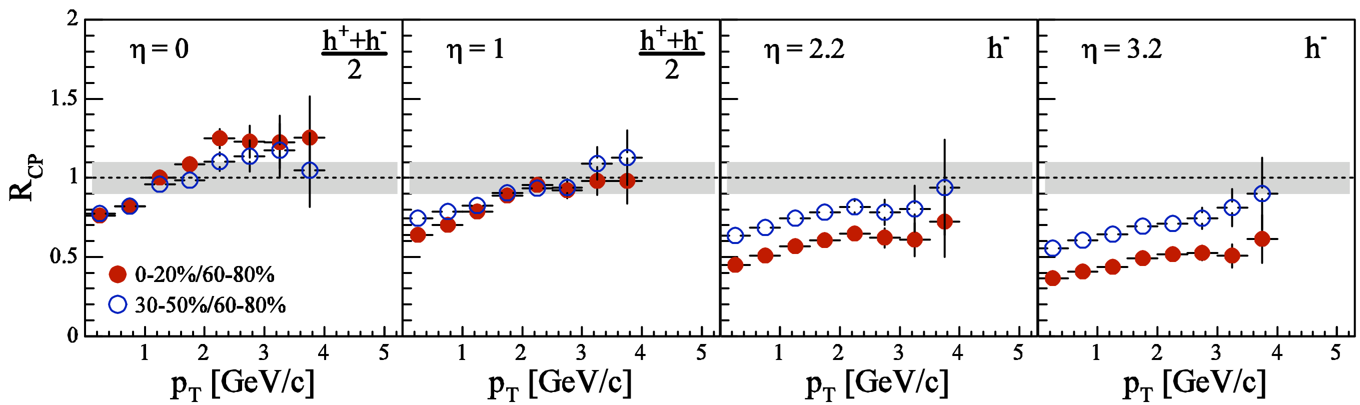
The plot is from D. Kharzeev, Yu. K., K. Tuchin '03
(see also Kharzeev, Levin, McLerran, '02 – original prediction, Albacete, Armesto, Kovner, Salgado, Wiedemann, '03)

R_{dAu} at different rapidities

R^{dAu}



R^{CP} – central to peripheral ratio

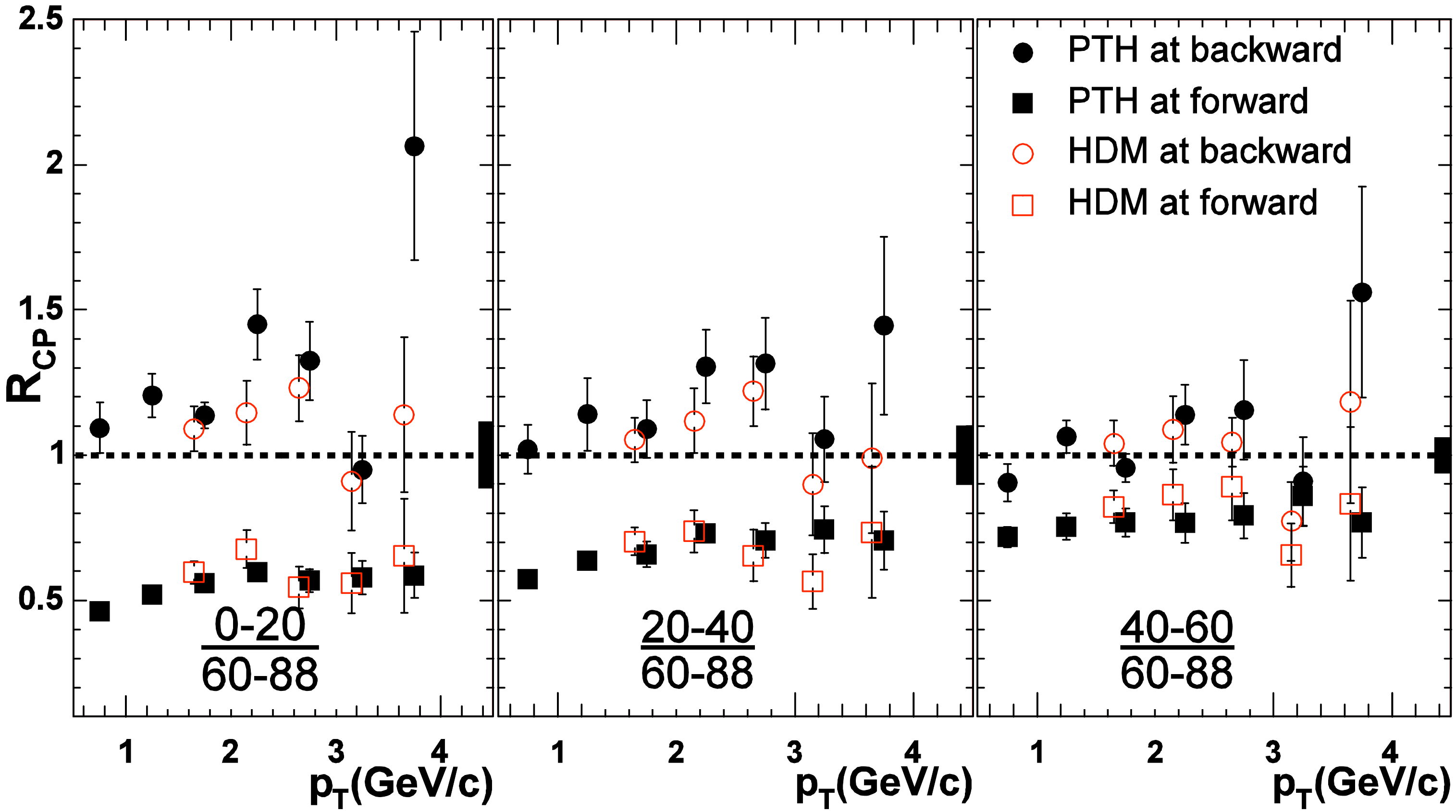


The data from BRAHMS Collaboration nucl-ex/0403005

Our prediction of suppression seems to be confirmed!

(indeed quarks have to be included too to describe the data)

R_{d+Au} at forward and backward rapidities

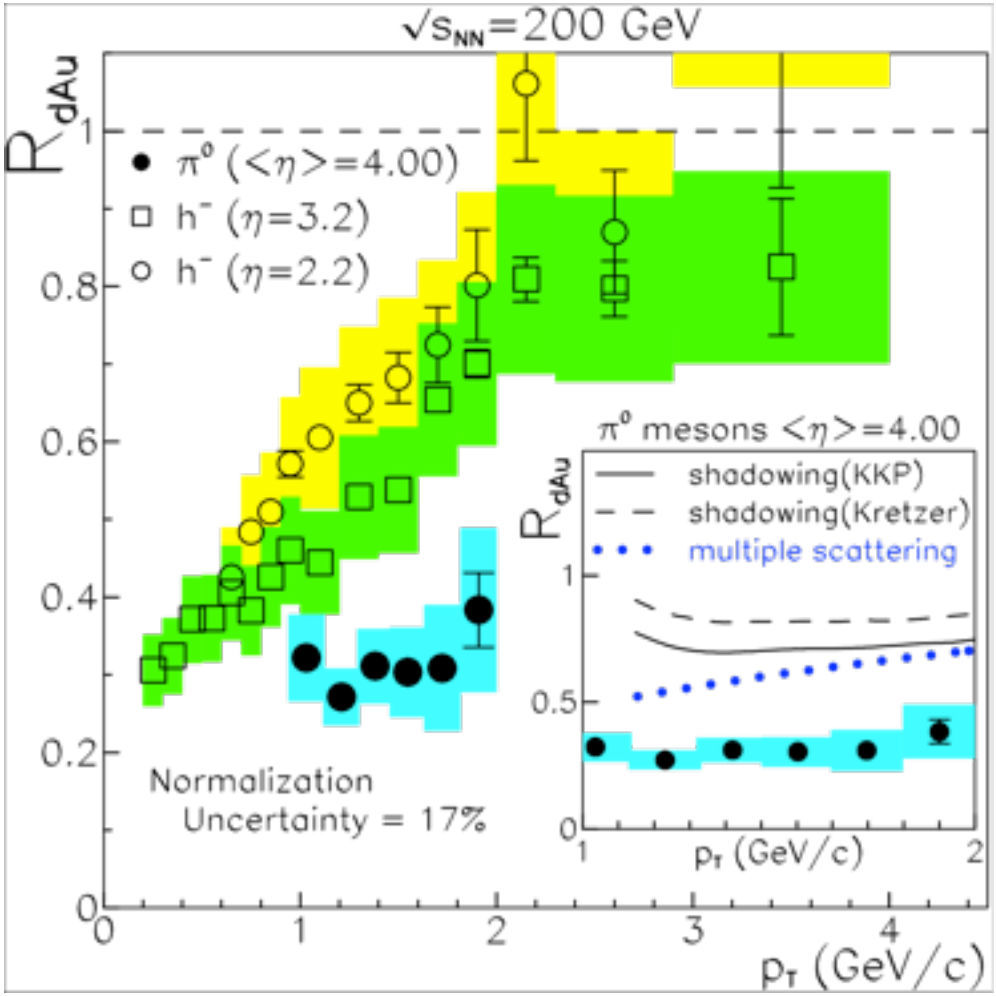


PHENIX data, nucl-ex/0411054

More Recent Data

Recent STAR data shows even stronger suppression at rapidity of 4.0, strengthening the case for CGC.

(figure from nucl-ex/0602011)

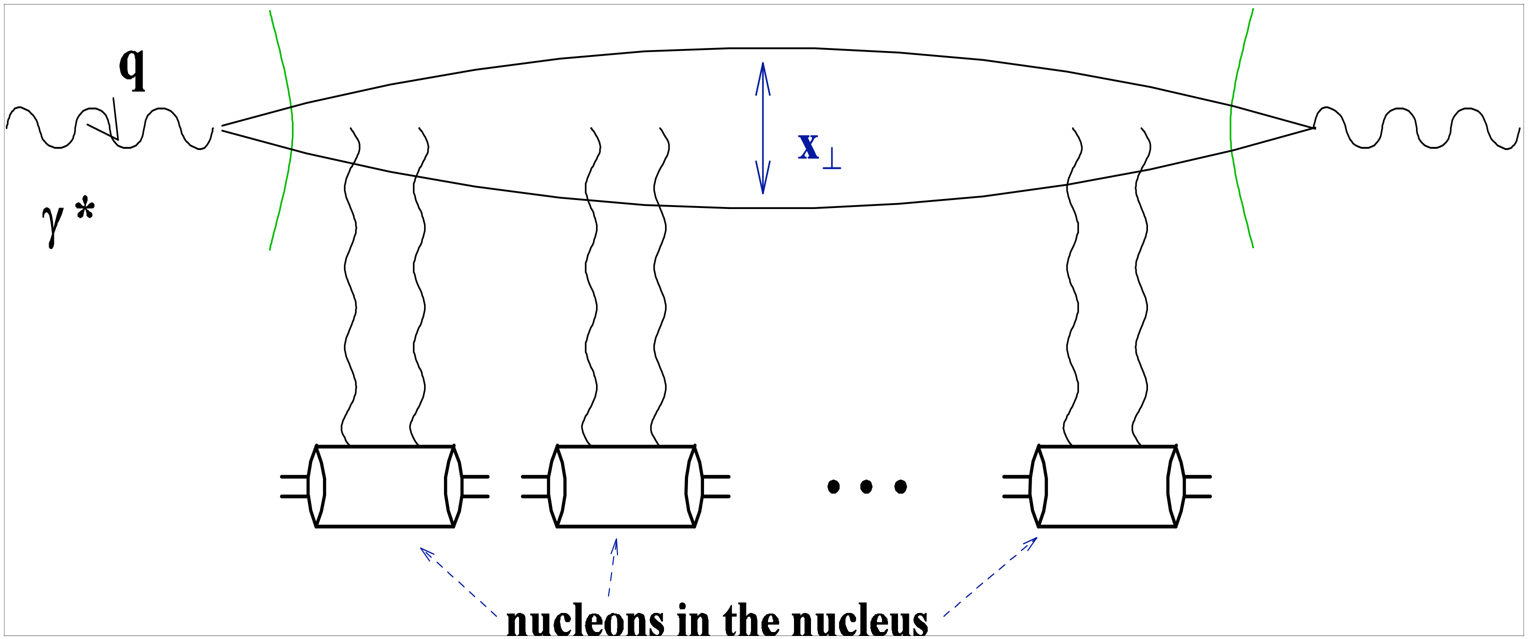


Recent Progress

A. Running Coupling

DIS in the Classical Approximation

The DIS process in the rest frame of the target is shown below. It factorizes into



$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = \Phi^{\gamma^* \to q\bar{q}} \otimes N(x_\perp, Y = \ln(1/x_{Bj}))$$

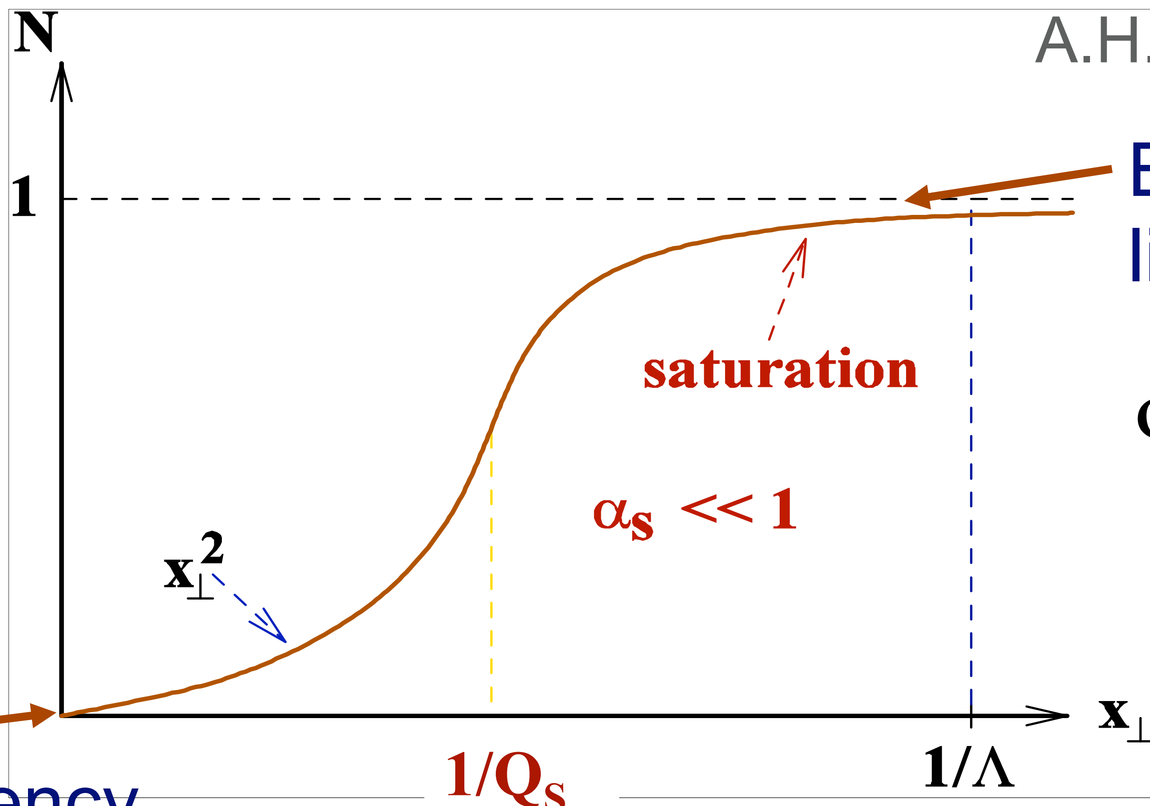
with rapidity Y=ln(1/x)

DIS in the Classical Approximation

The dipole-nucleus amplitude in the classical approximation is

$$N(x_\perp, Y) = 1 - \exp\left[- \frac{x_\perp^2 Q_S^2}{4} \ln \frac{1}{x_\perp \Lambda} \right]$$

A.H. Mueller, '90



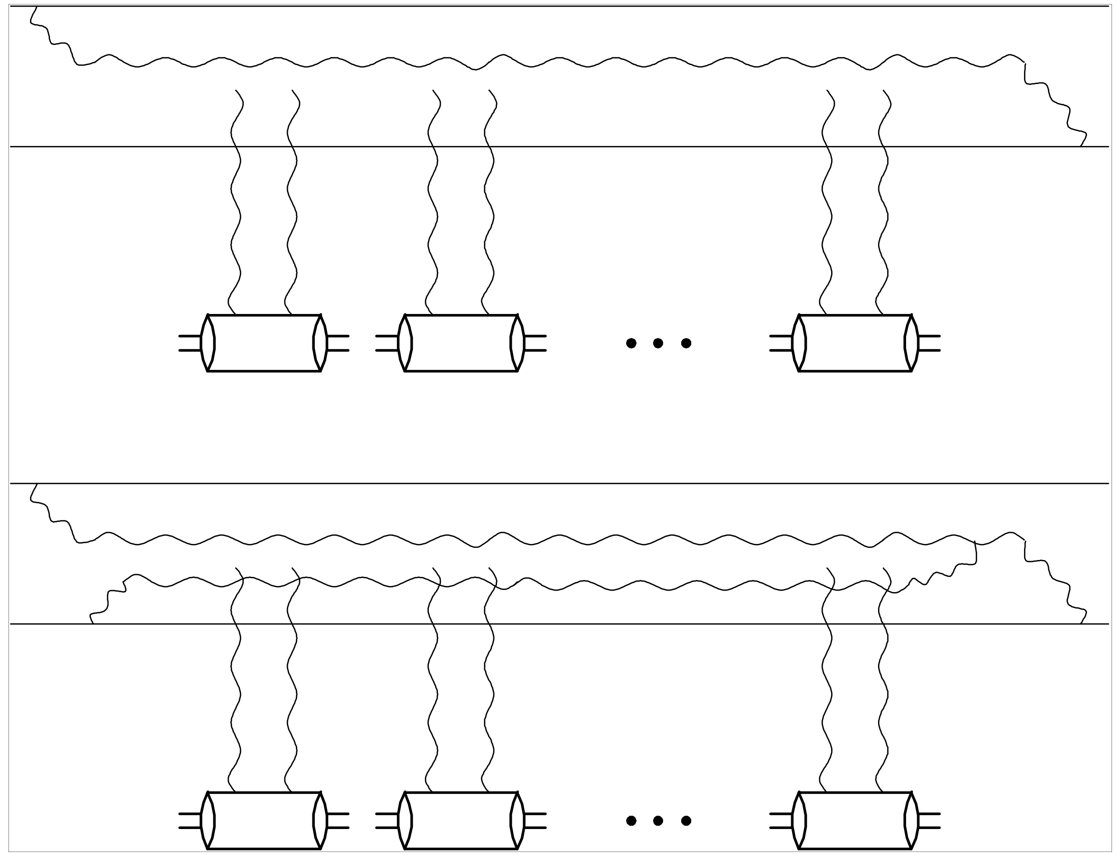
Black disk limit,

$$\sigma_{tot} < 2\pi R^2$$

Color transparency

Quantum Evolution

As energy increases the higher Fock states including gluons on top of the quark-antiquark pair become important. They generate a cascade of gluons.

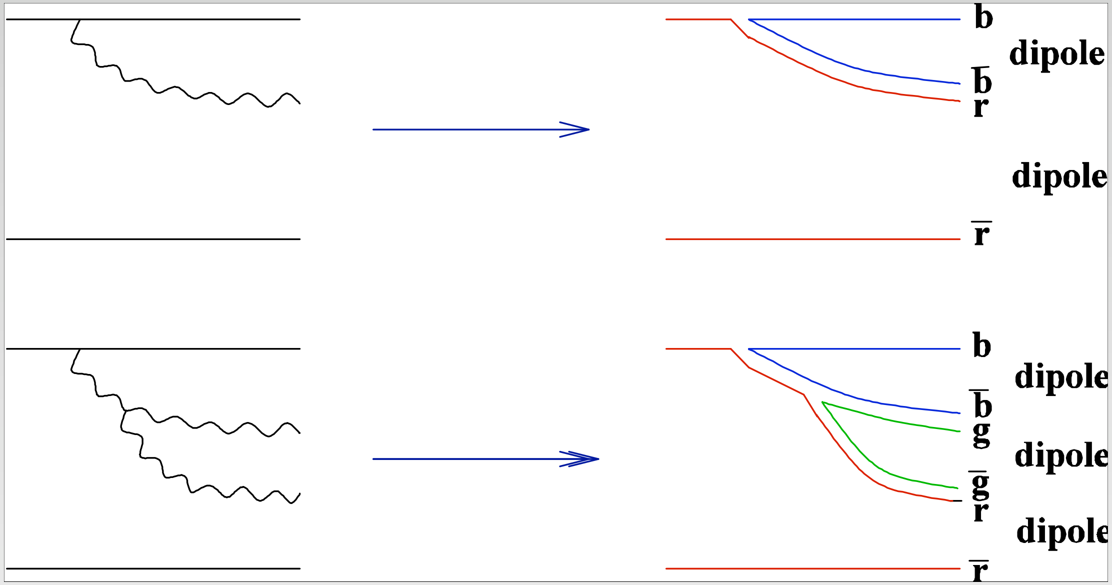


These extra gluons bring in powers of $\alpha_s \ln s$, such that when $\alpha_s << 1$ and $\ln s >> 1$ this parameter is $\alpha_s \ln s \sim 1$.

Resumming Gluonic Cascade

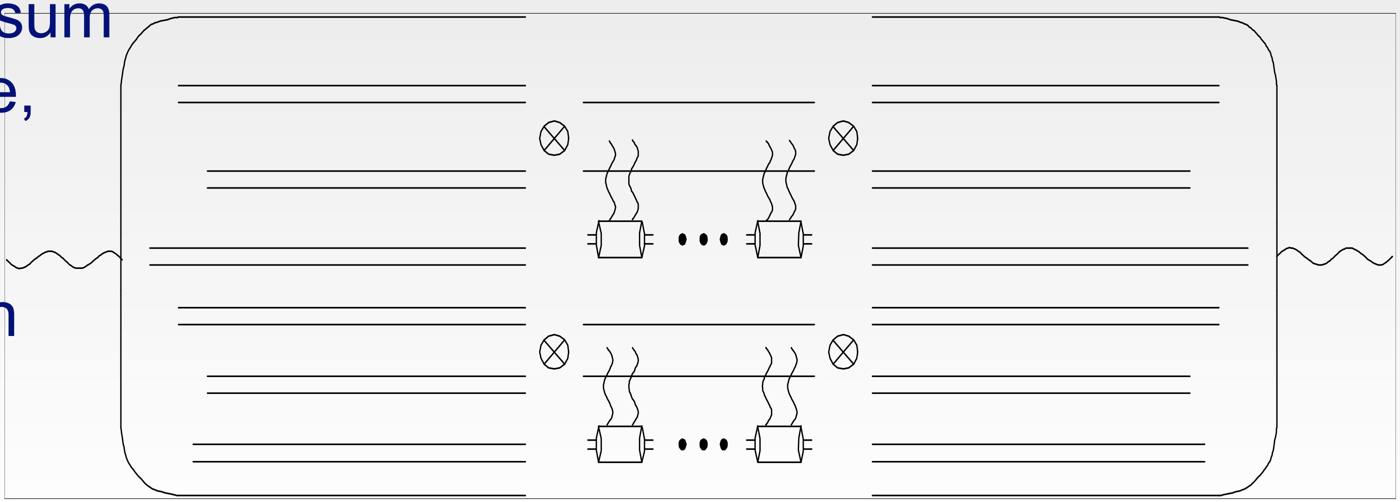
In the large-N_C limit of QCD the gluon corrections become color dipoles. Gluon cascade becomes a dipole cascade.

A. H. Mueller, '93-'94

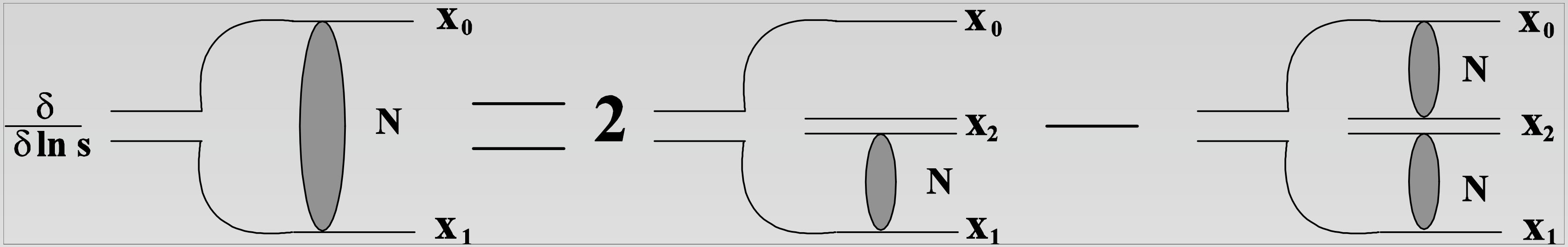


We need to resum dipole cascade, with each final state dipole interacting with the target.

Yu. K. '99



Nonlinear Evolution Equation



Defining rapidity Y=ln s we can resum the dipole cascade

$$\frac{\partial N(x_{01},Y)}{\partial Y} = \frac{\alpha_S N_C}{\pi^2}\int d^2x_2 \left[\frac{x_{01}^2}{x_{02}^2\, x_{12}^2} - 2\pi\,\delta^2(\underline{x}_{01}-\underline{x}_{02})\ln\left(\frac{x_{01}}{\rho}\right)\right] N(x_{02},Y)$$

$$-\frac{\alpha_S N_C}{2\pi^2}\int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2} N(x_{02},Y)\, N(x_{12},Y)$$

I. Balitsky, ’96, HE effective lagrangian
Yu. K., ’99, large N_C QCD

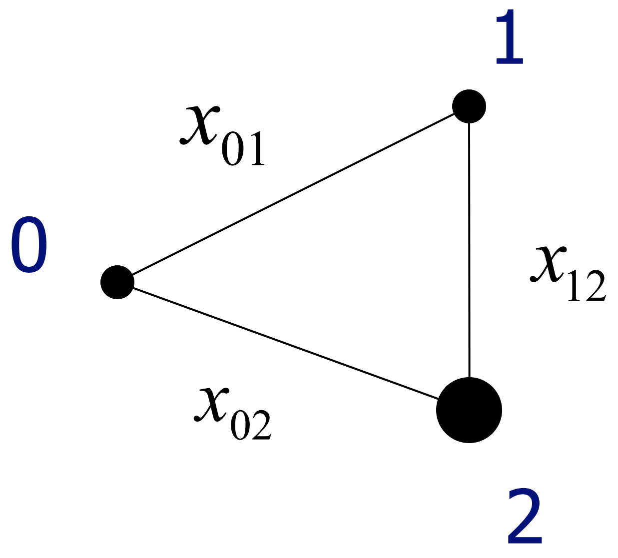
$$N(x_\perp, Y=0) = 1 - \exp\left[-\frac{x_\perp^2 Q_S^2}{4}\ln\frac{1}{x_\perp \Lambda}\right]$$

initial condition

⇨ Linear part is BFKL, quadratic term brings in damping

What Sets the Scale for the Running Coupling?

$$\frac{\partial\, N(x_0, x_1, Y)}{\partial\, Y} = \frac{\alpha_S\, N_C}{2\pi^2} \int d^2 x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\, N(x_2, x_1, Y)]$$



transverse
plane

What Sets the Scale for the Running Coupling?

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$
$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y) N(x_2, x_1, Y)]$$

$\alpha_S(???)$

In order to perform consistent calculations it is important to know the scale of the running coupling constant in the evolution equation.

There are three possible scales – the sizes of the "parent" dipole and "daughter" dipoles x_{01}, x_{21}, x_{20}. Which one is it?

Preview

- The answer is that the running coupling corrections come in as a **"triumvirate"** of couplings (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\boxed{\alpha_\mu \Rightarrow \frac{\alpha_S(\ldots)\,\alpha_S(\ldots)}{\alpha_S(\ldots)}}$$

cf. Braun '94, Levin '94

- The scales of three couplings are somewhat involved.

Results: Transverse Momentum Space

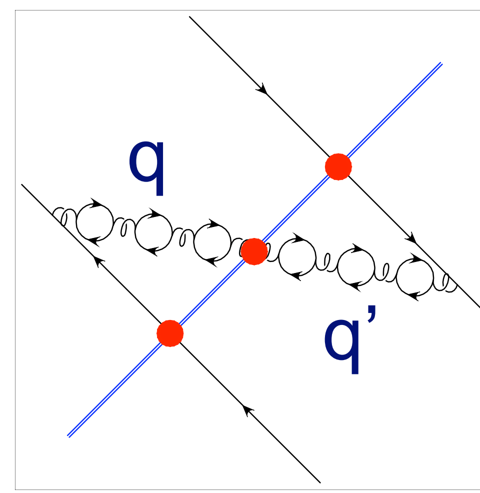
The resulting JIMWLK kernel with running coupling corrections is

$$\alpha_\mu\, K(\mathbf{x}_0, \mathbf{x}_1; \mathbf{z}) = 4 \int \frac{d^2q\, d^2q'}{(2\pi)^4}\, e^{-i\mathbf{q}\times(\mathbf{z}-\mathbf{x}_0) + i\mathbf{q}'\times(\mathbf{z}-\mathbf{x}_1)}\, \frac{\mathbf{q}\times\mathbf{q}'}{\mathbf{q}^2\, \mathbf{q}'^2}\, \frac{\alpha_S(\mathbf{q}^2)\,\alpha_S(\mathbf{q}'^2)}{\alpha_S(Q^2)}$$

where

$$\ln\frac{Q^2}{\mu^2} = \frac{\mathbf{q}^2 \ln(\mathbf{q}^2/\mu^2) - \mathbf{q}'^2 \ln(\mathbf{q}'^2/\mu^2)}{\mathbf{q}^2 - \mathbf{q}'^2} - \frac{\mathbf{q}^2\, \mathbf{q}'^2}{\mathbf{q}\times\mathbf{q}'}\, \frac{\ln(\mathbf{q}^2/\mathbf{q}'^2)}{\mathbf{q}^2 - \mathbf{q}'^2}$$

The BK kernel is obtained from the above by summing over all possible emissions of the gluon off the quark and anti-quark lines.



Running Coupling BK

Here's the BK equation with the running coupling corrections (H. Weigert, Yu. K. '06; I. Balitsky, '06):

$$\frac{\partial N(x_0, x_1, Y)}{\partial Y} = \frac{N_C}{2\pi^2} \int d^2 x_2$$

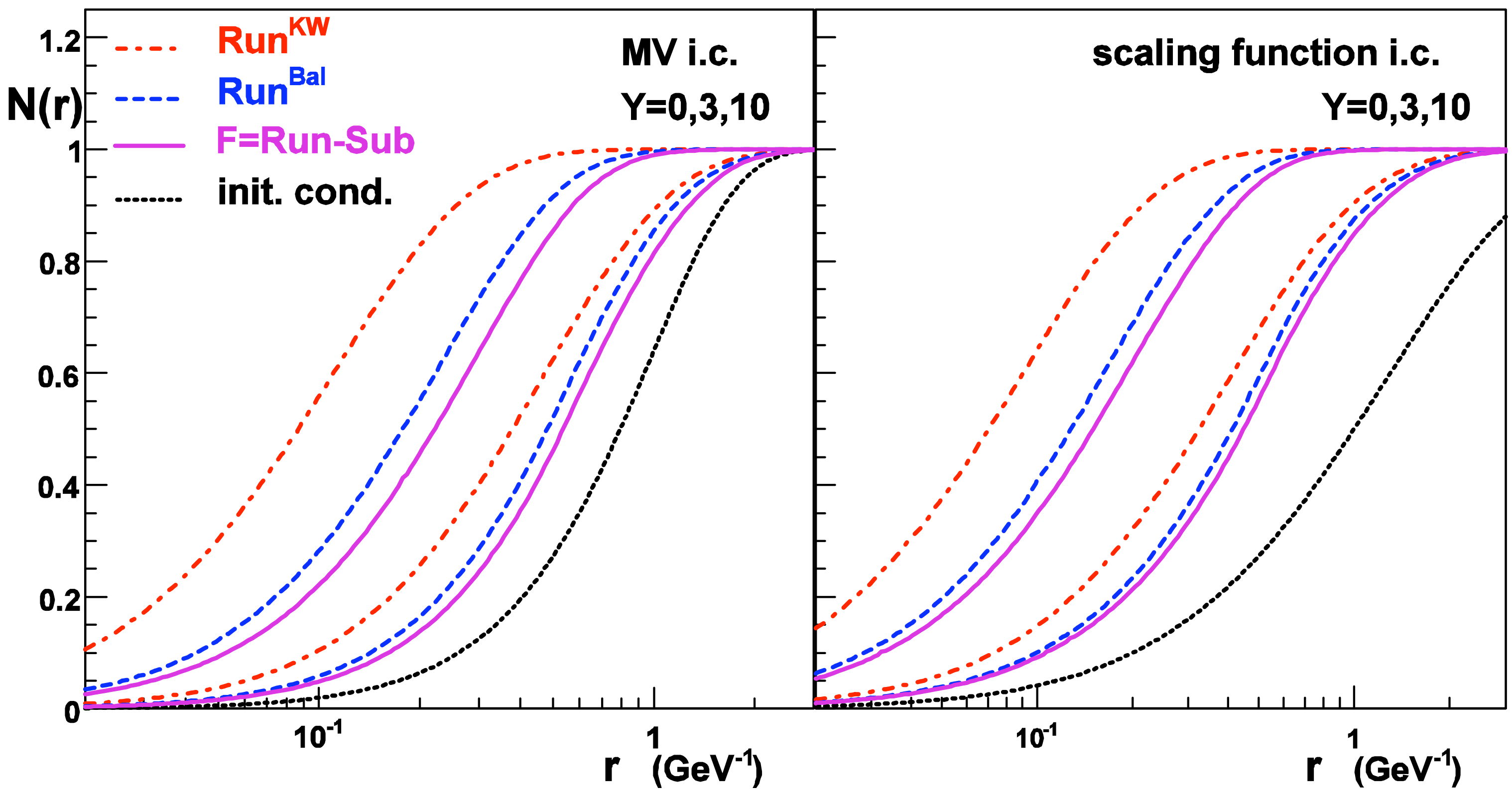
$$\times \left[\frac{\alpha_S(1/x_{02}^2)}{x_{02}^2} + \frac{\alpha_S(1/x_{12}^2)}{x_{12}^2} - 2\,\frac{\alpha_S(1/x_{02}^2)\,\alpha_S(1/x_{12}^2)}{\alpha_S(1/R^2)}\,\frac{\mathbf{x}_{20} \times \mathbf{x}_{21}}{x_{02}^2\, x_{12}^2} \right]$$

$$\times [N(x_0, x_2, Y) + N(x_2, x_1, Y) - N(x_0, x_1, Y) - N(x_0, x_2, Y)\, N(x_2, x_1, Y)]$$

where

$$\ln R^2 \mu^2 = \frac{x_{20}^2 \ln(x_{21}^2\, \mu^2) - x_{21}^2 \ln(x_{20}^2\, \mu^2)}{x_{20}^2 - x_{21}^2} + \frac{x_{20}^2\, x_{21}^2}{\mathbf{x}_{20} \times \mathbf{x}_{21}}\, \frac{\ln(x_{20}^2 / x_{21}^2)}{x_{20}^2 - x_{21}^2}$$

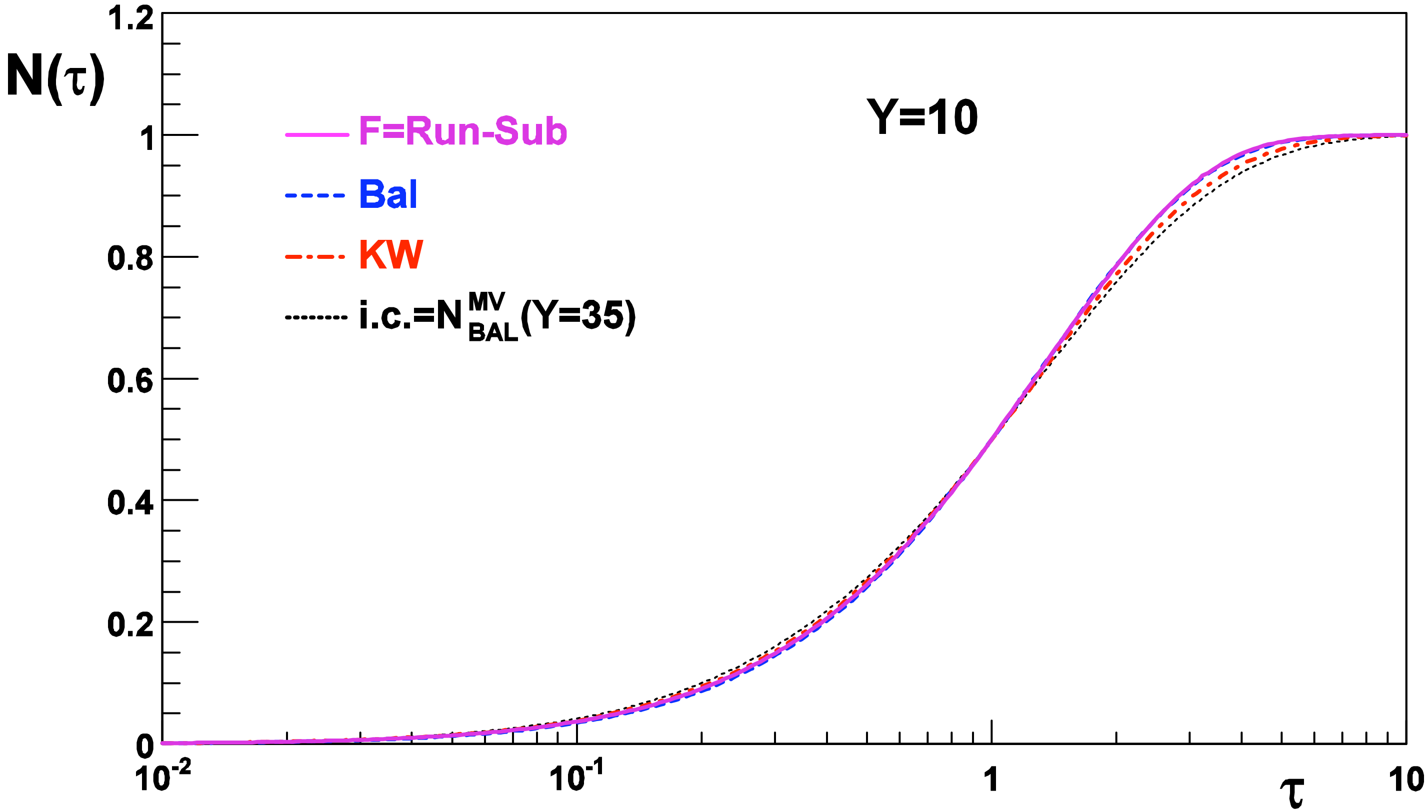
Solution of the Full Equation



Different curves – different ways of separating running coupling from NLO corrections. Solid curve includes all corrections.

J. Albacete, Yu.K. '07

Geometric Scaling



$$\tau = r\, Q_S(Y)$$

At high enough rapidity we recover geometric scaling, all solutions fall on the same curve. This has been known for fixed coupling: however, the shape of the scaling function is different in the running coupling case!

J. Albacete, Yu.K. '07

B. NLO BFKL/BK/JIMWLK

BFKL with Running Coupling

We can also write down an expression for the BFKL equation with running coupling corrections (H. Weigert, Yu.K. '06):

$$\frac{\partial \phi(k,Y)}{\partial Y} = \frac{N_c}{2\pi^2}\int d^2q \left\{ \frac{2}{(\mathbf{k}-\mathbf{q})^2}\alpha_S((\mathbf{k}-\mathbf{q})^2)\phi(q,Y) - \frac{k^2}{q^2(\mathbf{k}-\mathbf{q})^2}\frac{\alpha_S(q^2)\alpha_S((\mathbf{k}-\mathbf{q})^2)}{\alpha_S(k^2)}\phi(k,Y) \right\}$$

- cf. Braun '94, Levin '94

NLO BK/JIMWLK Evolution

- NLO BK/JIMWLK was calculated by Balitsky and Chrilli '07
- The answer is simple:

$$
\begin{aligned}
&\frac{d}{d\eta}\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\} \\
&= \frac{\alpha_s}{2\pi^2}\int d^2z\, \frac{(x-y)^2}{X^2Y^2}\Big\{1+\frac{\alpha_s}{4\pi}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \big(\frac{67}{9}-\frac{\pi^2}{3}\big)N_c - \frac{10}{9}n_f \\
&\qquad\qquad - 2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big]\Big\}\,[\mathrm{Tr}\{\hat U_x \hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z \hat U_y^\dagger\} - N_c\mathrm{Tr}\{\hat U_x \hat U_y^\dagger\}] \\
&+ \frac{\alpha_s^2}{16\pi^4}\int d^2z\,d^2z'\Big[\Big(-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]} \\
&+ \frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] + \frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big) \\
&\qquad\qquad \times [\mathrm{Tr}\{\hat U_x \hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z \hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'} \hat U_y^\dagger\} - \mathrm{Tr}\{\hat U_x \hat U_z^\dagger \hat U_{z'} U_y^\dagger \hat U_z \hat U_{z'}^\dagger\} - (z'\to z)] \\
&+ \Big\{\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big] - \frac{(x-y)^4}{X^2Y'^2X'^2Y^2}\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\,\mathrm{Tr}\{\hat U_x \hat U_z^\dagger\}\mathrm{Tr}\{\hat U_z \hat U_{z'}^\dagger\}\mathrm{Tr}\{\hat U_{z'} \hat U_y^\dagger\} \\
&+ 4n_f\Big\{\frac{4}{(z-z')^4} - 2\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2-)}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\mathrm{Tr}\{t^a\hat U_x t^b \hat U_y^\dagger\}[\mathrm{Tr}\{t^a \hat U_z t^b \hat U_{z'}^\dagger\} - (z'\to z)]\Big]
\end{aligned}
$$

NLO BK/JIMWLK

- It is known that NLO BFKL corrections are numerically large.
- Could it be that saturation effects make NLO BK/ JIMWLK corrections small?

Conclusions

- CGC/saturation physics tries to address fundamental and profound questions in strong interactions which have been around for over 40 years, longer than QCD itself.
- In recent decades small-x physics made significant theoretical progress: nonlinear BK and JIMWLK evolution equations have been written down which unitarize BFKL equation. Quasi-classical MV model was developed.
- Recent years saw much progress: running coupling corrections were found for small-x evolution equations: BFKL, BK and JIMWLK. NLO corrections to BK and JIMWLK have been calculated as well.
- CGC/saturation physics has enjoyed phenomenological success in describing DIS at HERA and RHIC d+Au and A+A data.